

Analysis on a Nambu–Jona-Lasinio Model of Dynamical Supersymmetry Breaking

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Abstract

This is a report on our newly proposed model of dynamical supersymmetry breaking with some details of the analysis involved. The model in the simplest version has only a chiral superfield (multiplet), with a strong four-superfield interaction in the Kähler potential that induces a real two-superfield composite with vacuum condensate. The latter has supersymmetry breaking parts, which we show to bear nontrivial solution following basically a standard nonperturbative analysis for a Nambu–Jona-Lasinio type model on a superfield setting. The real composite superfield has a spin one component but is otherwise quite unconventional. We discuss also the parallel analysis for the effective theory with the composite. Plausible vacuum solutions are illustrated and analyzed. The supersymmetry breaking solutions have generated soft mass(es) for the scalar avoiding the vanishing supertrace condition for the squared-masses of the superfield components. We also present some analysis of the resulted low energy effective theory with components of the composite become dynamical. The determinant of the fermionic modes is shown to be zero illustrating the presence of the expected Goldstino. The model gives the possibility of constructing a supersymmetric standard model with all (super)symmetry breaking masses generated dynamically and directly without the necessity of complicated hidden or mediating sectors.

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I. INTRODUCTION

With the discovery of the Higgs particle at the Large Hadron Collider (LHC), the full success of the Standard Model (SM) has been crowned. Unfortunately, we still do not see any clear indications of experimental features beyond so long as phenomenology at the TeV scale is concerned. Theorists are however mostly unsatisfied with the SM, particularly with its Higgs sector and explanation of the origin of the electroweak symmetry breaking. With a negative mass-square at the right scale put in by hand, the Higgs mechanism looks like only a phenomenological description of the ‘true’ theory behind. Moreover, the other parts of the SM theory have their field content tightly constrained by the gauge symmetry and no parameters with mass dimensions admissible; everything in the Higgs sector looks completely arbitrary in comparison. Another way of looking at the issue would be that the only natural value of any input mass parameter should be like the model cutoff scale. We need a model with a dynamical mechanism to generate the extra mass scale substantially below the cutoff.

Practical and experimentally accessible physics is really only about effective (field) theories. Taking the SM as an effective field theory, one would admit the higher dimensional operators with couplings suppressed by powers of the model cutoff scale in the Lagrangian. Actually, a dimension six term of four-fermion(/four-quark) interaction with otherwise strong coupling gives interesting nonperturbative dynamics that can break symmetries and generate masses [1]. That is the Nobel prize-winning classic Nambu–Jona-Lasinio (NJL) model [2], to which Higgs physics may correspond to the low energy effective theory with the Higgs doublet being identified as a two-fermion composite. This beautiful idea of the top-mode SM [3–8] fails to accommodate the too small phenomenological top quark mass [1]. At this point, it looks like a holomorphic supersymmetric version that gives the (minimal) supersymmetric standard model (SSM) with both Higgs supermultiplets as two-superfield composite maintains phenomenological viability [9].

The SSM is still the most popular candidate theory beyond the SM being matched to the LHC results. The theoretical beauty of supersymmetry is certain part of its appeal. The first supersymmetric Nambu–Jona-Lasinio (SNJL) model was introduced in the early eighties [10, 11], generalizing the four-fermion interaction to a four-superfield interaction of the same dimension in the Kähler potential. Recently, our group introduced the holo-

morphic version (HSNJL) as an alternative supersymmetrization [12] with a four-superfield interaction in the superpotential [9]. The two versions have different theoretical and phenomenological merits [9, 13, 14]. However, both versions require input soft supersymmetry breaking masses to have the dynamical (electroweak) symmetry breaking. On the other hand, the phenomenological SSM requires soft supersymmetry breaking masses/parameters the origin of which is typically depicted through elaborated constructions of complicated and contrived models with extra supersymmetry breaking and mediating sectors [15]. Under the background, it is the wish of us to find a simple model to get the supersymmetry breaking and soft mass generation dynamically, hopefully under a similar framework. That is essentially achieved. We just reported our first results of a new type of supersymmetric NJL model with a real two-superfield composite containing a spin one component. Following and extending the framework of our earlier analyses [13, 14], we have established that the model has the gap equation of the standard real soft mass parameter of the chiral superfield bearing nontrivial, hence supersymmetry breaking, solution when the four-superfield coupling is strong enough. The short letter we presented the results [16] only gives a sketch of the analyses involved and leaves the possibility of a more general supersymmetry breaking scenario not fully addressed. The current paper is to give a full account of all that.

In Sec. II, we present the model and the supergraph derivation of the superfield gap equation, elaborating carefully the extension of our framework of analysis [13, 14] with model parameters and correlation functions taken as superspace parameters, like constant superfields, containing supersymmetric and supersymmetry breaking parts. The superfield gap equation contains components which include wavefunction renormalization factor and two different soft mass parameters. In Sec. III, we discuss the effective theory picture with the composite and the matching effective potential analysis performed at the component field level, further strengthen the result and illustrate the physics involved. Sec. IV is devoted to analysis of the nontrivial, supersymmetry breaking solutions. In Sec. V, we go further to look at some dynamical features of the composite superfield or its various components at low energy, focusing on the Goldstino mode. Sec. VI is devoted to some further discussion of the supersymmetry breaking physics. Some remarks and conclusions will be presented in the last section. Two appendices are given, the first on some details of analytical expressions as background for the effective theory analysis and some results for two-point functions of the various components of the composite superfield relevant for their low energy dynamics, and

the second on propagator expressions for a (chiral) superfield and components admitting the most general mass parameters. The latter expressions have not been explicitly presented in the literature.

II. THE MODEL AND THE SUPERFIELD GAP EQUATION

The model has a dimension six four-superfield interaction similar but somewhat different from that of the SNJL model [10, 11, 16]. For the simplest example, we start with the single chiral superfield (multiplet) Lagrangian ¹

$$\mathcal{L} = \int d^4\theta \left[\Phi^\dagger \Phi + \frac{m_o}{2} \Phi \Phi \delta^2(\bar{\theta}) + \frac{m_o^*}{2} \Phi^\dagger \Phi^\dagger \delta^2(\theta) - \frac{g_o^2}{2} (\Phi^\dagger \Phi)^2 \right], \quad (1)$$

in which we have suppressed any multiplet (color) indices. We illustrate here a standard NJL gap equation analysis [10, 13, 14] applied to the soft supersymmetry breaking mass parameters, the brief result of which is reported in Ref.[16]. We are mostly interested in the generation of the usual soft supersymmetry breaking mass \tilde{m}^2 for the superfield Φ . Naively, if the bisuperfield condensate $\langle \Phi^\dagger \Phi|_D \rangle$ develops, we would have a soft supersymmetry breaking mass $g_o^2 \langle \Phi^\dagger \Phi|_D \rangle$. That is where our key interest is in.

Let us go onto a superfield gap equation analysis following and extending our earlier formulated framework [13, 14]. To implement an NJL-type gap equation analysis for the supersymmetry breaking, the first step of the self-consistent Hartree approximation is to add the interested soft mass term $-\int d^4\theta \Phi^\dagger \Phi \tilde{m}^2 \theta^2 \bar{\theta}^2$ to the free field part and re-subtract it as a mass-insertion type interaction. The formal gap equation is then given by

$$\tilde{m}^2 = \Sigma_{\tilde{m}}^{(loop)}(p) \Big|_{\text{on-shell}}, \quad (2)$$

where $\Sigma_{\tilde{m}}(p)$ is the two-point proper vertex for the scalar component A of Φ , as shown in the Fig. 1. Note that the four-superfield interaction, after the $d^4\theta$ integration, has the part $-g_o^2 A A^\dagger (\Phi \Phi^\dagger)|_{\theta, \bar{\theta}=0}$. We have also performed the calculation fully in the component field framework for case of Ref.[16], but prefer to illustrate the superfield calculation here in accordance with the formulation under the perspective discussed in Ref.[13]. We consider a superfield two-point proper vertex $\Sigma_{\Phi\Phi^\dagger}(p; \theta^2, \bar{\theta}^2)$ as taking value like a constant superfield

¹ Our basic notation is in line with that of Wess and Bagger [17].

$$\Phi \xrightarrow{\tilde{m}^2 \theta^2 \bar{\theta}^2} \Phi^\dagger + \left[\text{Diagram: a circle with } \Phi\Phi^\dagger \text{ above it, connected to a horizontal line with } \Phi \text{ on the left and } \Phi^\dagger \text{ on the right, with a dot at the connection point} \right] \theta^2 \bar{\theta}^2 = 0$$

FIG. 1: The soft mass gap equation in terms of the scalar component A of the superfield.

with components explicitly dependent on θ^2 and $\bar{\theta}^2$. The $\Sigma_{\tilde{m}}(p)$ of interest is then to be taken essentially as the $\theta^2 \bar{\theta}^2$ component of $\Sigma_{\Phi\Phi^\dagger}(p; \theta^2, \bar{\theta}^2)$. We have then a full superfield analog of the gap equation involving the latter, including also the constant component and the θ^2 component (with its conjugate). Potentially, one sees more interesting result options, like nontrivial solution from the θ^2 part of the full superfield gap equation would give an alternative option of supersymmetry breaking.

Before getting into our formulation, some comments on the symmetry issues are in order. Apart from supersymmetry itself, the model Lagrangian has, independent of the multiplet content of Φ_a , a $U(1)_R$ symmetry under which Φ_a has unit charge. With vanishing m_o , it has a full $U(N)$ symmetry under which the multiplet can be considered in the fundamental representation. The $m_o = 0$ case is really a main focus for us, though we do not enforce it in the analysis to keep our result more general. It is important to note that a nonzero mass is not necessary for our key result here, as presented below. The usual $1/N$ approximation picture, however, can still be valid with the mass nonzero. Φ_a may then be considered as an $SO(N)$, instead of $SU(N)$, multiplet. In both cases, there is also $U(1)$ Φ -number symmetry in the Lagrangian which is only violated by the mass term. In the naive case of really a single superfield, the gap equation analysis here would correspond to the quenched planar approximation of QED by Bardeen *et.al.* [18–20], which is commonly believed to give the correct qualitative result in the kind of dynamical symmetry breaking studies. Some more discussion of the issue in a somewhat different setting is available in Ref.[14].²

² For taking explicitly the single field case, there is then no difference in the four-superfield interaction considered here comparing to the old SNJL model. However, there are still a few key difference in our study compared to that. First, the coupling has different signs. Second, we ask and answer a different question. We ask the question if a $\bar{\Phi}\Phi$ composite and condensate may form in the absence of supersymmetry breaking terms in the original Lagrangian. The SNJL work asked if a kind of $\Phi\Phi$ composite and condensate may

To keep notation simple, we will present our analysis here onwards with the index suppressed, as if we are working on a single superfield. What we have in mind is really a N -multiplet of the $SO(N)$ or $SU(N)$. To retrieve result for a nontrivial N is straightforward. The one-loop contribution such as the one in $\Sigma_{\Phi\Phi^\dagger}(p; \theta^2, \bar{\theta}^2)$ or $\Sigma_{\tilde{m}}(p)$ will have to be multiplied by the factor N .

In the full superfield picture, $\Sigma_{\Phi\Phi^\dagger}(p; \theta^2, \bar{\theta}^2)$ should expand as

$$\Sigma_{\Phi_R\Phi_R^\dagger}(p; \theta^2\bar{\theta}^2) = \Sigma_r(p) - \Sigma_{\tilde{\eta}}(p)\theta^2 - \bar{\Sigma}_{\tilde{\eta}^*}(p)\bar{\theta}^2 - \Sigma_{\tilde{m}}(p)\theta^2\bar{\theta}^2. \quad (3)$$

The part $\Sigma_{\tilde{m}}$ in itself is like a proper self-energy contribution to the scalar but not the fermion component, hence soft supersymmetry breaking. With $\Phi = A + \sqrt{2}\psi\theta + F\theta^2$, it is a AA^* vertex. The soft supersymmetry breaking mass \tilde{m}^2 as a superfield term is just the $\theta^2\bar{\theta}^2$ component of the kinetic term, to which $\Sigma_{\Phi\Phi^\dagger}(p; \theta^2, \bar{\theta}^2)$ is the quantum correction to the latter. The part $\Sigma_{\tilde{\eta}}$ is somewhat less obvious. It is a proper vertex of AF^* , to be matched to another mass parameter $\tilde{\eta}$; the $\tilde{\eta}AF^*$ term gives another kind of soft supersymmetry breaking mass not usually discussed in the literature.³ Lastly, the supersymmetric part Σ_r gives only a kinetic term, hence contributes to wavefunction renormalization. It is then easy to appreciate that a consistent superfield treatment of the standard NJL analysis should consider modifying the superfield propagator to incorporate plausible nonperturbative parameter of the generic form given by

$$\mathcal{Y} = y - \tilde{\eta}_o\theta^2 - \tilde{\eta}_o^*\bar{\theta}^2 - \tilde{m}_o^2\theta^2\bar{\theta}^2 \quad (4)$$

containing not only the \tilde{m}^2 part but also its supersymmetric partners. We write here \tilde{m}_o^2 instead of \tilde{m}^2 as the parameter is not the physical soft mass yet. The component y contributes a (supersymmetry) wavefunction renormalization factor which renormalizes all mass parameters accordingly, as shown below explicitly. Notice that generation of nontrivial y breaks no symmetry while generation of \tilde{m}^2 breaks only supersymmetry. Nonvanishing $\tilde{\eta}$ however breaks the $U(1)_R$ symmetry together with supersymmetry.

form and got a sure no answer when there is no input supersymmetry breaking terms.

³ Looking at the content of the superfield kinetic term, one sees that it is the parameter for a AF^* term. After elimination of the auxiliary component, we have in general $|\tilde{\eta}|^2$ as an extra contribution to the scalar mass. In the presence of superfield mass m , or rather $\mathcal{M} = m - \eta\theta^2$, products of $\tilde{\eta}m$ and $\tilde{\eta}\eta$ may contribute to other mass terms. We do not consider any nonzero η here. Note that m , or \mathcal{M} with zero η , is only an input parameter of the original supersymmetric Lagrangian. One can easily see that $\tilde{\eta}m$ contributes a AA mass term after the elimination of the auxiliary component F , giving mass-squared eigenvalues of $|m|^2 + |\tilde{\eta}|^2 + \tilde{m}^2 \pm 2|\tilde{\eta}||m|$ to the two (real) scalar states.

To proceed with the derivation of the superfield gap equation, we add and subtract the term $\mathcal{Y}\bar{\Phi}\Phi$ and split the Lagrangian as $\mathcal{L} = \mathcal{L}_o + \mathcal{L}_{int}$ where

$$\mathcal{L}_o = \int \bar{\Phi}\Phi(1 + \mathcal{Y}) + \frac{m_o}{2}\Phi^2\delta^2(\bar{\theta}) + \frac{m_o^*}{2}\bar{\Phi}^2\delta^2(\theta) \quad (5)$$

and

$$\mathcal{L}_{int} = \int -\mathcal{Y}\bar{\Phi}\Phi - \frac{g_o^2}{2}\bar{\Phi}\Phi\bar{\Phi}\Phi, \quad (6)$$

in which we have left the $d^4\theta$ implicit. To restore the canonical kinetic term in the presence of a plausibly nonzero y , we introduce the renormalized superfield $\Phi_R \equiv \sqrt{Z}\Phi = \sqrt{1+y}\Phi$ which gives

$$\mathcal{L}_o = \int \bar{\Phi}_R\Phi_R(1 - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2) + \frac{m}{2}\Phi_R^2\delta^2(\bar{\theta}) + \frac{m^*}{2}\bar{\Phi}_R^2\delta^2(\theta). \quad (7)$$

The mass parameters are of course renormalized ones, to be divided by the wavefunction renormalization parameter Z ; explicitly $m = \frac{m_o}{1+y}$, for example. The quantum effective action is

$$\begin{aligned} \Gamma = & \bar{\Phi}_R\Phi_R(1 - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2) + \frac{m}{2}\Phi_R^2\delta^2(\bar{\theta}) + \frac{m^*}{2}\bar{\Phi}_R^2\delta^2(\theta) \\ & - \mathcal{Y}_R\bar{\Phi}_R\Phi_R - \frac{g^2}{2}\bar{\Phi}_R\Phi_R\bar{\Phi}_R\Phi_R + \Sigma_{\Phi_R\Phi_R^\dagger}\bar{\Phi}_R\Phi_R + \dots, \end{aligned} \quad (8)$$

where $g^2 = \frac{g_o^2}{(1+y)^2}$ is the renormalized four-superfield coupling and \mathcal{Y}_R is similarly given by

$$\mathcal{Y}_R = \frac{\mathcal{Y}}{Z} = \frac{y}{1+y} - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2. \quad (9)$$

The superfield gap equation under the NJL framework is then given by

$$-\mathcal{Y}_R + \Sigma_{\Phi_R\Phi_R^\dagger}^{(loop)}(p; \theta^2\bar{\theta}^2) \Big|_{\text{on-shell}} = 0; \quad (10)$$

in component form, we have

$$\begin{aligned} \frac{y}{1+y} &= \Sigma_r^{(loop)}(p) \Big|_{\text{on-shell}}, \\ \tilde{\eta} &= \Sigma_{\tilde{\eta}}^{(loop)}(p) \Big|_{\text{on-shell}}, \\ \tilde{m}^2 &= \Sigma_{\tilde{m}^2}^{(loop)}(p) \Big|_{\text{on-shell}}, \end{aligned} \quad (11)$$

where in accordance of the standard NJL analysis one uses the one-loop contribution to $\Sigma_{\Phi_R\Phi_R^\dagger}(p; \theta^2\bar{\theta}^2)$ from the four-superfield interaction. The diagrammatic illustration of the

$$\Phi_R \text{---} \text{X} \text{---} \Phi_R^\dagger \quad + \quad \Phi_R \text{---} \text{loop} \text{---} \Phi_R^\dagger = 0$$

FIG. 2: The renormalized superfield gap equation , with $\mathcal{Y}_R = \frac{y}{1+y} - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2$.

renormalized superfield gap equation is given in Fig. 2. We can see that the naive expectation of Eq.(2) works, so long as it understood to be applied to the superfield and couplings with the wavefunction renormalization factor properly incorporated. However, the wavefunction renormalization factor itself can be retrieved from a gap equation. Note that results reported in Ref.[16] corresponds to assuming $\tilde{\eta}$ remains zero from the beginning, which will be shown to be a consistent solution; the gap equation figure therein is the $\theta^2\bar{\theta}^2$ part of the one here.

We perform a supergraph calculation for $\Sigma_{\Phi_R\Phi_R^\dagger}(p; \theta, \bar{\theta})$ directly. The relevant superfield propagator is given by

$$\begin{aligned} \langle T(\Phi(1)_R\Phi_R^\dagger(2)) \rangle &= \frac{-i}{p^2 + |m|^2} \delta_{12}^4 - i \frac{\tilde{\eta}(Q - 2|m|^2)}{Q^2 - 4|m|^2|\tilde{\eta}|^2} \theta_1^2 \delta_{12}^4 - i \frac{\tilde{\eta}^*(Q - 2|m|^2)}{Q^2 - 4|m|^2|\tilde{\eta}|^2} \bar{\theta}_1^2 \delta_{12}^4 \\ &+ i \frac{(\tilde{m}^2 + |\tilde{\eta}|^2)Q - 4|m|^2|\tilde{\eta}|^2}{(p^2 + |m|^2)(Q^2 - 4|m|^2|\tilde{\eta}|^2)} \left[\frac{D_1^2 \theta_1^2 \bar{\theta}_1^2 \bar{D}_1^2}{16} \right] \delta_{12}^4 \\ &+ i \frac{(-p^2|\tilde{\eta}|^2 + \tilde{m}^2|m|^2)Q + 4p^2|m|^2|\tilde{\eta}|^2}{(p^2 + |m|^2)(Q^2 - 4|m|^2|\tilde{\eta}|^2)} \theta_1^2 \bar{\theta}_1^2 \delta_{12}^4, \end{aligned} \quad (12)$$

where $Q = p^2 + |m|^2 + |\tilde{\eta}|^2 + \tilde{m}^2$ and $\delta_{12}^4 = \delta^4(\theta_1 - \theta_2)$. The necessary evaluation of $\Sigma_{\Phi_R\Phi_R^\dagger}^{(loop)}(p; \theta^2\bar{\theta}^2) \Big|_{\text{on-shell}}$ is much to similar previous cases [13]. The result is given by

$$\begin{aligned} \Sigma_{\Phi_R\Phi_R^\dagger}^{(loop)}(p; \theta^2\bar{\theta}^2) \Big|_{\text{on-shell}} &= -g^2 \int^E \left[\frac{1}{k^2 + |m|^2} + \frac{\tilde{\eta}(Q_k - 2|m|^2)}{Q_k^2 - 4|m|^2|\tilde{\eta}|^2} \theta^2 + \frac{\tilde{\eta}^*(Q_k - 2|m|^2)}{Q_k^2 - 4|m|^2|\tilde{\eta}|^2} \bar{\theta}^2 \right. \\ &\quad - \frac{(\tilde{m}^2 + |\tilde{\eta}|^2)Q_k - 4|m|^2|\tilde{\eta}|^2}{(k^2 + |m|^2)(Q_k^2 - 4|m|^2|\tilde{\eta}|^2)} (1 - k^2\theta^2\bar{\theta}^2 + 4k_a\sigma_{\alpha\dot{\alpha}}^a\theta^\alpha\bar{\theta}^{\dot{\alpha}}) \\ &\quad \left. - \frac{(-k^2|\tilde{\eta}|^2 + \tilde{m}^2|m|^2)Q_k + 4k^2|m|^2|\tilde{\eta}|^2}{(k^2 + |m|^2)(Q_k^2 - 4|m|^2|\tilde{\eta}|^2)} \theta^2\bar{\theta}^2 \right], \end{aligned} \quad (13)$$

where the \int^E denotes integration over Euclidean four-momentum k with the measure $\frac{d^4k}{(2\pi)^4}$ and $Q_k = k^2 + |m|^2 + |\tilde{\eta}|^2 + \tilde{m}^2$. Each of the five terms in the above expression comes exactly from the corresponding term in the superfield propagator. The $4k_a\sigma_{\alpha\dot{\alpha}}^a\theta^\alpha\bar{\theta}^{\dot{\alpha}}$ term vanishes

upon integration. The others can be pull together to give the component gap equations as

$$\begin{aligned}
\frac{y}{1+y} &= \Sigma_r^{(loop)}(p) \Big|_{\text{on-shell}} = -g^2 \int^E \frac{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2} , \\
\tilde{\eta} &= \Sigma_{\tilde{\eta}}^{(loop)}(p) \Big|_{\text{on-shell}} = g^2 \tilde{\eta} \int^E \frac{(k^2 - |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2} , \\
\tilde{m}^2 &= \Sigma_{\tilde{m}^2}^{(loop)}(p) \Big|_{\text{on-shell}} = g^2 \int^E \frac{1}{(k^2 + |m|^2)} \frac{1}{(k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - 4|m|^2|\tilde{\eta}|^2} \\
&\quad \cdot \{ [\tilde{m}^2(k^2 - |m|^2) + 2k^2|\tilde{\eta}|^2] (k^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) - 8k^2|m|^2|\tilde{\eta}|^2 \} . \quad (14)
\end{aligned}$$

Nontrivial solutions of the three coupled equations with nonvanishing $\tilde{\eta}$ and/or \tilde{m}^2 give supersymmetry breaking solutions. We postpone the analysis of the nontrivial solution till after the discussion of the effective theory picture in the next section. Note that nontrivial y value gives wavefunction renormalization to Φ which does not change the qualitative answer to if supersymmetry breaking solution with the soft mass generation exists. Our analysis will explicitly demonstrate that.

III. THE EFFECTIVE THEORY PICTURE

Following the general effective theory picture of the NJL-type models, we modify the model Lagrangian by adding to it

$$\mathcal{L}_s = \int d^4\theta \frac{1}{2} (\mu U + g_o \bar{\Phi} \Phi)^2 , \quad (15)$$

where U is an ‘auxiliary’ real superfield and mass parameter μ taken as real and positive (for $g_o^2 > 0$). The equation of motion for U , from the full Lagrangian $\mathcal{L} + \mathcal{L}_s$ gives

$$U = -\frac{g_o}{\mu} \bar{\Phi} \Phi , \quad (16)$$

showing it as a superfield composite of $\bar{\Phi}$ and Φ . The condition says the model with $\mathcal{L} + \mathcal{L}_s$ is equivalent to that of \mathcal{L} alone. Expanding the term in \mathcal{L}_s , we have a cancellation of the dimension six interaction in the full Lagrangian, giving it as

$$\mathcal{L}_{eff} \equiv \mathcal{L} + \mathcal{L}_s = \int d^4\theta \left[\bar{\Phi} \Phi + \frac{\mu^2}{2} U^2 + \mu g_o U \bar{\Phi} \Phi + \frac{m_o}{2} \Phi^2 \delta(\bar{\theta}) + \frac{m_o^*}{2} \bar{\Phi}^2 \delta(\theta) \right] . \quad (17)$$

Obviously, if $U|_D$ develops a vacuum expectation value (VEV), supersymmetry is broken spontaneously and the superfield Φ gains a soft supersymmetry breaking mass of $\tilde{m}_o^2 = -\mu g_o \langle U|_D \rangle$. The above looks very much like the standard features of NJL-type model.

Notice that while U does contain a vector component, its couplings differ from that of the usually studied ‘vector superfield’ which is a gauge field supermultiplet. That is in addition to having μ as like a supersymmetric mass for U , which can be compatible only with a broken gauge symmetry. As such, model with superfield U is not usually discussed. The superfield can be seen as two parts, as illustrated by the following component expansion,

$$U(x, \theta, \bar{\theta}) = \frac{C(x)}{\mu} + \sqrt{2}\theta\frac{\chi(x)}{\mu} + \sqrt{2}\bar{\theta}\frac{\bar{\chi}(x)}{\mu} + \theta\theta\frac{N(x)}{\mu} + \bar{\theta}\bar{\theta}\frac{N^*(x)}{\mu} + \sqrt{2}\theta\sigma^\mu\bar{\theta}v_\mu(x) + \sqrt{2}\theta\theta\bar{\theta}\bar{\lambda}(x) + \sqrt{2}\bar{\theta}\bar{\theta}\theta\lambda(x) + \theta\theta\bar{\theta}\bar{\theta}D(x), \quad (18)$$

where the components C , χ , and N is the first part which has the content of like a chiral superfield with however C being real. The μ factor is put to set the mass dimensions right. The rest is like the content of a superfield for the usual gauge field supermultiplet, with D and v_μ real. The effective Lagrangian in component form is given by

$$\begin{aligned} \mathcal{L}_{eff} = & (1 + g_o C) [A^* \square A + i(\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \psi + F^* F] + \frac{m_o}{2} (2AF - \psi\psi) + \frac{m_o^*}{2} (2A^* F^* - \bar{\psi}\bar{\psi}) \\ & + \mu CD - \mu\chi\lambda - \mu\bar{\chi}\bar{\lambda} + NN^* - \frac{\mu^2}{2} v^\nu v_\nu - \mu g_o \psi \lambda A^* - \mu g_o \bar{\psi} \bar{\lambda} A + \mu g_o D A^* A \\ & - i \frac{g_o}{2} \bar{\psi} \bar{\sigma}^\mu \chi \partial_\mu A + i \frac{g_o}{2} (\partial_\mu \bar{\psi}) \bar{\sigma}^\mu \chi A - g_o \chi \psi F^* + g_o N A F^* \\ & + i \frac{g_o}{2} \bar{\chi} \bar{\sigma}^\mu \psi \partial_\mu A^* - i \frac{g_o}{2} A^* \bar{\chi} \bar{\sigma}^\mu \partial_\mu \psi - g_o \bar{\chi} \bar{\psi} F + g_o N^* A^* F \\ & - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu i A^* \partial_\nu A + \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu i (\partial_\nu A^*) A - \frac{\mu g_o}{\sqrt{2}} \eta^{\mu\nu} v_\mu \bar{\psi} \bar{\sigma}_\nu \psi. \end{aligned} \quad (19)$$

Notice that like F , N and D have mass dimension two.

Under the $U(1)_R$ symmetry, A and F have charge $+1$ and -1 . The superfield U is uncharged. However, components N , χ and λ carry nontrivial $U(1)_R$ charges -2 , -1 and $+1$, respectively. For the $m_o = 0$ case, there is an extra $U(1)$ Φ -number symmetry with common charge for all components. All components of U is not charged under the latter.

In accordance with the ‘quark-loop’ approximation in the (standard) NJL gap equation analysis and our particular supergraph calculation scheme above in particular, we consider plausible nontrivial vacuum solution with nonzero vacuum expectation values (VEVs) for the composite scalars C , D and N . While N is complex, we can safely taken $n \equiv \langle N \rangle$ to be real here. At least we can exploit the $U(1)_R$ symmetry to absorb any phase at the expense of having a complex m_o the phase of which does not show up in the calculation. First note that scalar C couples to kinetic terms of components of Φ ; $c \equiv \langle C \rangle$ hence contributes to a supersymmetric wavefunction renormalization of the latter. It is the supersymmetric

part of $\Sigma_{\Phi\Phi^\dagger}^{(loop)}(p; \theta^2, \bar{\theta}^2)$ an unavoidable part of the one-loop supergraph in our gap equation calculation in the previous section. Again, we should go to the renormalized superfield $\Phi_R = \sqrt{(1 + g_o c)} \Phi$ in the following calculations, with renormalized mass m and coupling g . With $n \equiv \langle N \rangle$ and $d \equiv \langle D \rangle$, we have $-gn$ and $-\mu gd$ corresponding to the supersymmetry breaking masses $\tilde{\eta}$ and \tilde{m}^2 of Φ_R . In the former case, it gives a $A_R F_R^*$ component term. Note that $\langle N \rangle$ is the only VEV that breaks the $U(1)_R$ symmetry, as C and D carry no charges, though both $\langle N \rangle$ and $\langle D \rangle$ break supersymmetry.

With propagators for the components of the renormalized ‘quark’ superfield Φ_R as given in the appendix, one can easily obtain the minimum condition for the effective potential following the Weinberg tadpole method [21, 22]. Firstly, for C -tadpoles, we have a Φ_R loop or in component form one from each of A_R , ψ_R , and F_R . Hence, we have up to one loop level

$$\Gamma_C^{(1)} = \Gamma_C^{(1)\text{tree}} + \Gamma_{C_A}^{(1)} + \Gamma_{C_\psi}^{(1)} + \Gamma_{C_F}^{(1)} = \mu d - g I_C , \quad (20)$$

where

$$\begin{aligned} I_C &= I_{CA} - 2I_{C\psi} + I_{CF} ; \\ I_{CA} &= \int^E \frac{k^2(k^2 + |m|^2 + g^2|n|^2 - \mu gd)}{(k^2 + |m|^2 + g^2|n|^2 - \mu gd)^2 - 4g^2|n|^2|m|^2} , \\ I_{C\psi} &= \int^E \frac{k^2}{k^2 + |m|^2} , \\ I_{CF} &= \int^E \frac{(k^2 - \mu gd)(k^2 + |m|^2 + g^2|n|^2 - \mu gd)}{(k^2 + |m|^2 + g^2|n|^2 - \mu gd)^2 - 4g^2|n|^2|m|^2} . \end{aligned} \quad (21)$$

Next, the N^* -tadpole is given by

$$\Gamma_{N^*}^{(1)} = n - g I_N , \quad (22)$$

where

$$I_N = \int^E \frac{gn(k^2 - |m|^2 + g^2|n|^2 - \mu gd)}{(k^2 + |m|^2 + g^2|n|^2 - \mu gd)^2 - 4g^2|n|^2|m|^2} . \quad (23)$$

The D -tadpole is given by

$$\Gamma_D^{(1)} = \mu c + \mu g I_D \quad (24)$$

where

$$I_D = \int^E \frac{k^2 + |m|^2 + g^2|n|^2 - \mu gd}{(k^2 + |m|^2 + g^2|n|^2 - \mu gd)^2 - 4g^2|n|^2|m|^2} . \quad (25)$$

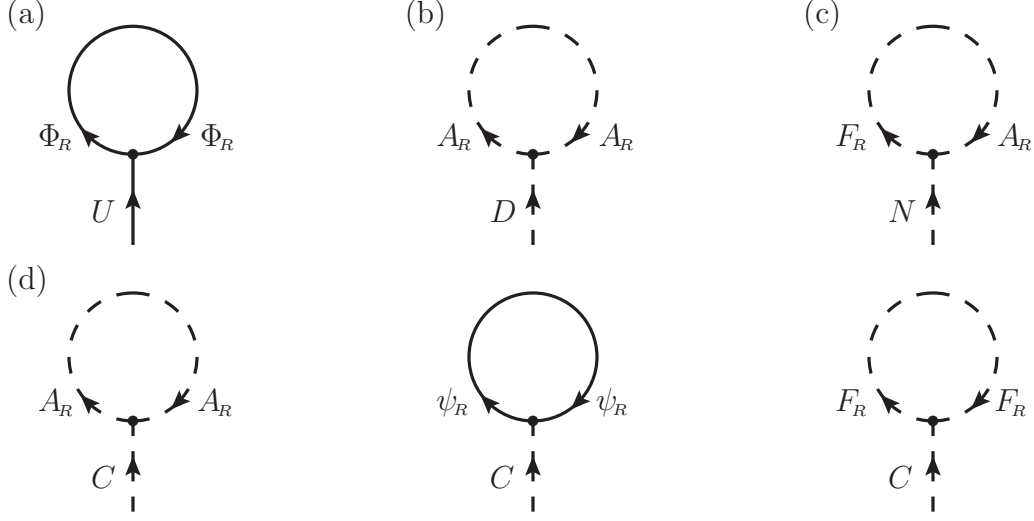


FIG. 3: The tadpole diagrams: a) the superfield diagram; b) D-tadpole; c) N-tadpole; d) C-tadpoles.

The tadpole diagrams are illustrated in Fig. 3. We look for vacuum solution with $-\Gamma_a^{(1)} \equiv \partial V(c, n, d)_{1\text{-loop}}/\partial a = 0$ for $a = c, n, d$. Firstly, note that the vanishing of N^* -tadpole is equivalent to

$$n(1 - g^2 I_{N'}) = 0, \quad (26)$$

with $I_{N'}$ given by $I_N = gnI_{N'}$; vanishing D -tadpole gives

$$c = -gI_D; \quad (27)$$

the vanishing C -tadpole condition is

$$\mu d = gI_C. \quad (28)$$

To get the physics picture clear, one can identify the soft masses generated for the superfield Φ by $\tilde{\eta} = -gn$ and $\tilde{m}^2 = -\mu gd$. We will explore nontrivial solutions for the soft masses below.

It is interesting to see that the effective potential analysis for (the components of) the composite superfield U can be shown directly to be equivalent to the superfield gap equation, which we illustrated explicitly in Ref.[16] and duplicated here. In terms of the superfield, the potential minimum condition is given by

$$\mu^2 \langle U \rangle + U_{\text{tadpole}} = 0 \quad \implies \quad \mu g \langle U \rangle = -g^2 I_{\Phi_R \Phi_R^\dagger}^{(\text{loop})} \quad (29)$$

where $I_{\Phi_R\Phi_R^\dagger}^{(loop)}$ is the momentum integral of the $\Phi_R\Phi_R^\dagger$ propagator loop (*cf.* the first diagram in Fig 3). Note that from the original Lagrangian with two-superfield composite assumed, we can obtain $-g^2\langle(\Phi_R\Phi_R^\dagger)\rangle = \mathcal{Y}_R$, which is equivalent to $\mu g\langle U\rangle = \mathcal{Y}_R = \Sigma_{\Phi_R\Phi_R^\dagger}^{(loop)}(p; \theta^2\bar{\theta}^2)\Big|_{\text{on-shell}} = -g^2 I_{\Phi_R\Phi_R^\dagger}^{(loop)}$. The same loop integral is of course involved in both the gap equation picture and the effective potential analysis. The results here are in direct matching with the corresponding discussion for the NJL case presented in Ref.[11], though for a superfield theory instead. The component field effective potential analysis here above hence really serves as a double-check of the superfield gap equation analysis of the previous section. In terms of component fields, we need the soft mass identifications above as well as $y = g_o c$, or $\frac{y}{1+y} = gc$.

IV. SUPERSYMMETRY BREAKING SOLUTIONS

Let us pull together the gap equation in terms of $y = (1 - gc)^{-1}$, $\tilde{\eta}(= -gn)$ and $\tilde{m}^2(= -\mu gd)$. We have

$$\tilde{\eta}(1 - g^2 I'_N) = 0 \quad \text{and} \quad \tilde{m}^2 = -g^2 I_C ,$$

as a set of coupled equations to be solved simultaneously as the integrals are complicated expressions involving the two soft mass parameters. The third equation of

$$y = (1 + g^2 I_D)^{-1}$$

independently gives the y value for any solution of $\tilde{\eta}$ and \tilde{m} . One can easily check that the equations are indeed identical to the set of Eq.(14) derived from the original Lagrangian through the supergraph evaluation. Note that the y parameter does not correspond to any physical quantity and hence may be considered of little interest. The case of zero soft masses is consistent, as I_C vanishes in the supersymmetric limit. The point of interest is if solutions of nontrivial supersymmetry breaking masses $\tilde{\eta}$ and \tilde{m} exist.

The first soft mass gap equation gives $g^2 I'_N = 1$ for nontrivial $\tilde{\eta}$, for the case of which we have

$$I'_N = \frac{1}{2} \left[\left(1 - \frac{|m|}{|\tilde{\eta}|}\right) I_F(m_{A_-}^2) + \left(1 + \frac{|m|}{|\tilde{\eta}|}\right) I_F(m_{A_+}^2) \right] , \quad (30)$$

where $I_F(S) [\equiv \int^E \frac{1}{k^2 + S}]$ has been used to denote integral of the Feynman propagator for

field of mass square S and we have the scalar mass eigenvalues ⁴

$$m_{A_{\mp}}^2 = \tilde{m}^2 + (|m| \mp |\tilde{\eta}|)^2. \quad (31)$$

Similarly, we have

$$I_C = - \left(m_{A_-}^2 - \frac{\tilde{m}^2}{2} \right) I_F(m_{A_-}^2) - \left(m_{A_+}^2 - \frac{\tilde{m}^2}{2} \right) I_F(m_{A_+}^2) + 2|m|^2 I_F(|m|^2). \quad (32)$$

If we take $m = 0$, we would have

$$I'_N \longrightarrow I_F(|\tilde{\eta}|^2 + \tilde{m}^2)$$

and

$$I_C \longrightarrow -\tilde{m}^2 I_F(|\tilde{\eta}|^2 + \tilde{m}^2) - 2|\tilde{\eta}|^2 I_F(|\tilde{\eta}|^2 + \tilde{m}^2).$$

The second soft mass gap equation becomes

$$g^2 I_F(|\tilde{\eta}|^2 + \tilde{m}^2) \left(1 + 2 \frac{|\tilde{\eta}|^2}{\tilde{m}^2} \right) = 1 \quad (33)$$

which is not compatible with the first one ($g^2 I'_N = 1$) unless $\tilde{\eta} = 0$. It remains to see if there exists $\tilde{\eta} \neq 0$ solution for some nonzero values of m . After some algebra, one can rewrite the solution equations in the form

$$g^2 I_F(m_{A_{\mp}}^2) = \frac{|m|\tilde{m}^2 \mp 2|\tilde{\eta}|(|m| \pm |\tilde{\eta}|)^2}{|m|(2|m|^2 - 2|\tilde{\eta}|^2 + \tilde{m}^2)} + \frac{2|m|(|m| \pm |\tilde{\eta}|)}{2|m|^2 - 2|\tilde{\eta}|^2 + \tilde{m}^2} g^2 I_F(|m|^2). \quad (34)$$

The two equations have the same form with only the $|\tilde{\eta}|$ variable come in different signs. And both reduces to the same equation for the $I_F(m_A)$ at the $|\tilde{\eta}| = 0$ limit, which is the gap equation for the limiting case [16]. Evaluating the integrals with model cutoff Λ , with all variables and parameters casted in terms of dimensionless counterparts normalized to Λ given by $G = \frac{g^2 \Lambda^2}{16\pi^2}$, $s = \frac{\tilde{m}^2}{\Lambda^2}$, and $t = \frac{|m|^2}{\Lambda^2}$, the two equations are equivalent to

$$\begin{aligned} \frac{1}{G(s, t, z)} = & \frac{s + 2tz(1 - z)}{s + 2tz(1 - z)^2} + \frac{2t(1 - z)}{s + 2tz(1 - z)^2} t \ln \left[1 + \frac{1}{t} \right] \\ & - \frac{s + 2t(1 - z^2)}{s + 2tz(1 - z)^2} [s + t(1 + z)^2] \ln \left[1 + \frac{1}{s + t(1 + z)^2} \right], \end{aligned} \quad (35)$$

⁴ In connection to the scalar masses, it is interesting to note that nontrivial $\tilde{\eta}$ also gives spontaneous CP violation. Though we keep m as a complex parameter in our analysis, its complex phase in the original Lagrangian is not physical and can be taken away. The original Lagrangian hence conserves CP. Or as seen here, presence of nonzero $m\tilde{\eta}$ product splits the masses of the scalar and pseudoscalar part of A and produces mass mixing between them, giving the mass eigenvalues. The surprising part is one only needs nonzero $m\tilde{\eta}$ to have it, even real value would do.

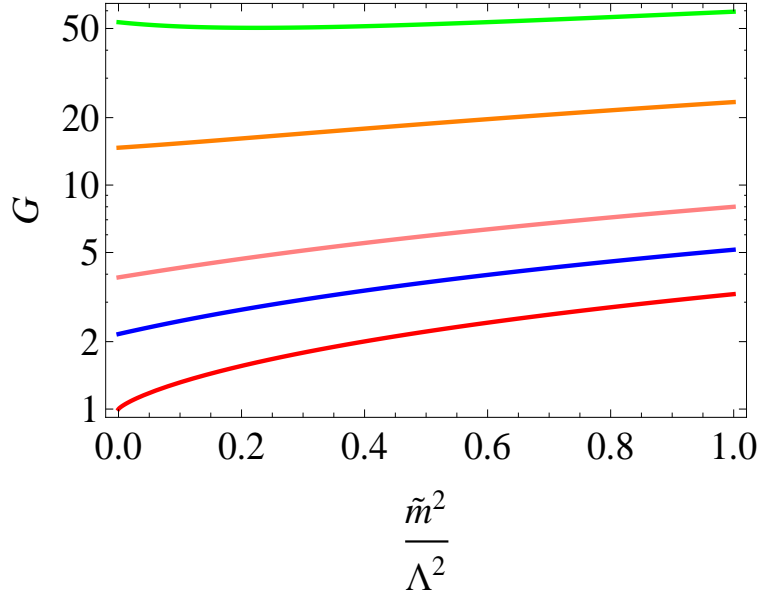


FIG. 4: Numerical plot of nontrivial solutions to the soft mass gap equation with $|\tilde{\eta}| = 0$. Coupling parameter $G = \frac{Ng^2\Lambda^2}{16\pi^2}$ is plotted against the normalized soft mass parameter $s \left(= \frac{\tilde{m}^2}{\Lambda^2} \right)$ for $t \left(= \frac{|m|^2}{\Lambda^2} \right)$ values of 0 (red), 0.1 (blue), 0.2 (pink), 0.4 (orange), 0.5 (green), from the lowest to the highest curves, respectively. Here N is the ‘color’ factor for the case of the basic chiral superfield Φ being an $SO(N)$ or $SU(N)$ multiplet not shown explicitly in the calculation, and Λ is the model cutoff scale. Notice that the critical coupling increases from $G = 1$ for nonzero values of the input supersymmetric mass m . (Figure duplicated from Ref.[16].)

for $z = \mp \frac{|\tilde{\eta}|}{|m|}$ respectively. We need simultaneous solutions for s and z for reasonable values of model parameters G and t . The two equations for positive and negative (but equal) values of z of course collapse to one at $z = 0$, which is the vanishing $|\tilde{\eta}|$ solutions which we presented in Ref.[16]. We duplicate the illustrating plots here in Fig. 4.

Actually, in the $\tilde{\eta} = 0$ ($z = 0$) case, all the above integrals simplify analytically. In particular, we have

$$I_D \longrightarrow I_F(|m|^2 + \tilde{m}^2)$$

and

$$I_C \longrightarrow -(\tilde{m}^2 + 2|m|^2)I_F(|m|^2 + \tilde{m}^2) + 2|m|^2 I_F(|m|^2)$$

(I'_N is irrelevant). The masses in the Feynman propagators correspond to the scalar and fermion masses. It is interesting to note that for $m = 0$ ($t = 0$), we have the simple

result $g^2 I_F(\tilde{m}^2) = 1$ ⁵, which is the same as the basic NJL model one except with the soft mass \tilde{m}^2 replacing the (Dirac) fermionic mass (see for example Ref.[11]) if we take $\frac{g^2}{2}$ as the four-fermion coupling in the model. For more details, we see that solutions for nontrivial \tilde{m}^2 for the case is given by the reduced form of Eq.(35) as $\frac{1}{G} = 1 - s \ln \left[1 + \frac{1}{s} \right]$ obviously giving solution for $0 < s < 1$ for the strong enough coupling $G > 1$. It can be seen from the numerical plot that the value of the \tilde{m}^2 solution rises fast with increasing G . However, nonzero t has a strong limiting effect. It increases the critical coupling needed for a nontrivial solution to s very substantially. In fact, taking the limit $s \rightarrow 0$, the equation becomes $\frac{1}{G_c} = 1 + \frac{2t}{t+1} - 3t \ln \left(1 + \frac{1}{t} \right)$, which gives the critical coupling G_c as a function of t . It can be seen then as t increases from zero, $\frac{1}{G_c}$ decreases and reaches vanishing value (*i.e.* $G_c \rightarrow \infty$) at a critical t value of about 0.55, beyond which no coupling G will be strong enough to break the supersymmetry and generate the soft mass. This part of the results has been reported in Ref.[16].

Looking for solution with nontrivial $\tilde{\eta}$ is more tricky and requires a very careful analysis scanning the numerical results. Again, we check plots of the effective coupling G as given in Eq.(35) versus s simultaneously for positive and negative values of z of fixed magnitude, at a fixed input t value. Numerically, where the two curves (dubbed G_+ and G_- , respectively) intersect within the window of interest gives a solution. One only have then to numerically scan the plots of the G_+ and G_- curves to see all the solutions. The window of interest is restricted to positive G value and $0 < s \leq 1$ plus the extra constraint of both of the mass eigenvalues of the scalar states in Φ to be within the cutoff Λ [*cf.* Eq.(31)]. This is the generalization of $s \leq 1$ to the nontrivial $\tilde{\eta}$, $|z| \neq 0$ case. The constraint is given by

$$s + t(1 + |z|)^2 \leq 1. \quad (36)$$

It is strong. For any t value, it first restricts $|z|$ of interest to $\leq \frac{1}{\sqrt{t}} - 1$. Close to the upper limit means s admissible has to be very small. So, the constraint may cut out quite a range, if not all, of the s value of interest. We find that solution exists in general, though some of the features of the solution locations are not somewhat peculiar and not easy to understand.

⁵ It is interesting to note that in the case we have the gap equation for the renormalization factor, which is equivalent to the vanishing D -tadpole condition $c = -gI_D$, giving $c = -gI_F(\tilde{m}^2)$ hence $g_o c = -.5$. The wavefunction renormalization factor is $Z = 1 + g_o c = .5$, of order one but tangible. That is a clear indication of the nonperturbative nature of the results and that there is nothing improper in the analysis.

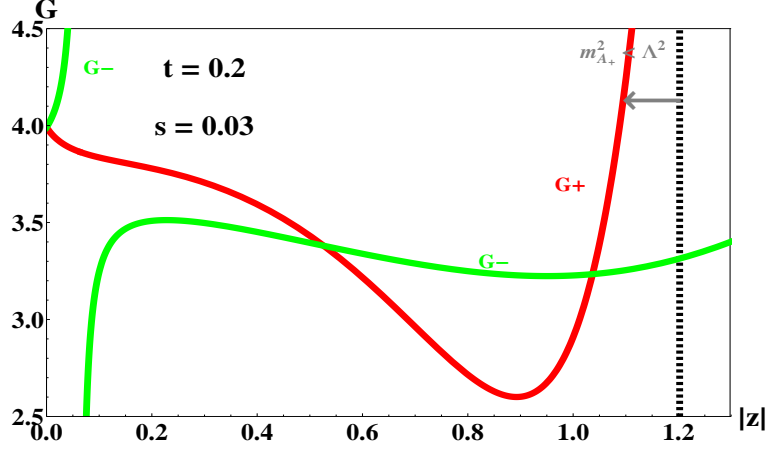


FIG. 5: An illustrative of intersecting point solutions, with G versus $|z|$.

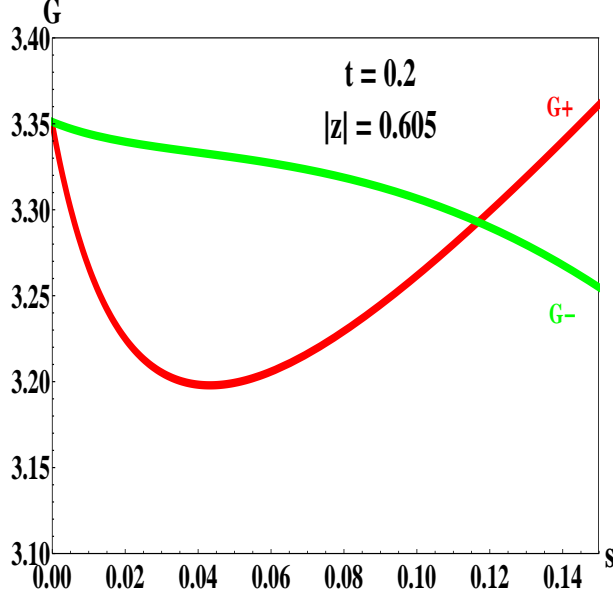


FIG. 6: Illustrative intersecting point solution plots, with G versus s .

We scanned on the effective coupling G versus s , $|z|$, and t plots to study the behavior of the intersecting point solutions and check for consistence. The results are as follow: For somewhat large t , solution exists only at large enough $|z|$, for example the minimal $|z|$ value for solution at $t = .3$ is about 1. Such solution certainly violates (36). Actually, solution satisfying the constraint shows up only for t below about .265, which also guarantees the

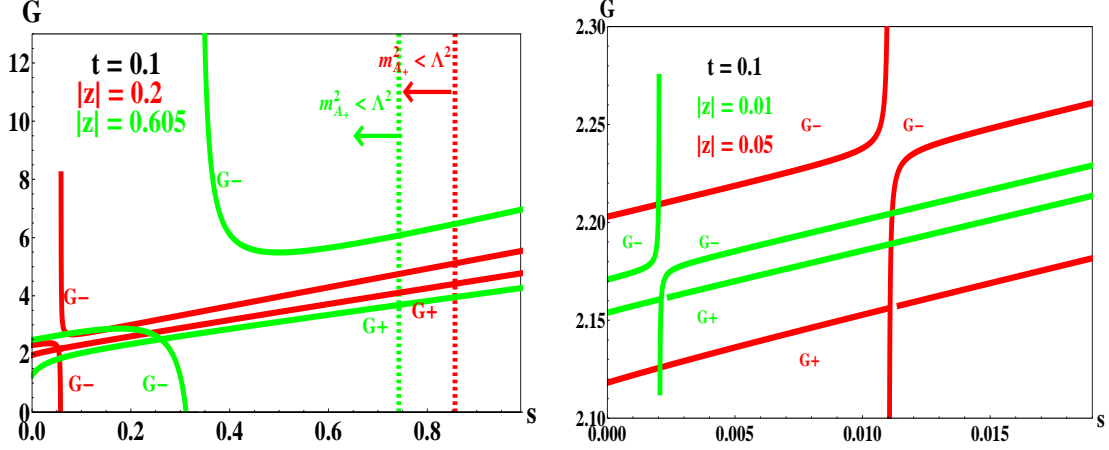


FIG. 7: Illustrative intersecting point solution plots, with G versus s .; two cases in each frame for comparison. The two colors each corresponds to the case of one set of fixed parameter values as shown. Intersecting points of G^+ and G^- curves of the same color give the solution point for the value of $|z|$.

G_+ curve to be smooth at least within the numerical window of interest. Moreover, the G versus $|z|$ plots for any t and s essentially always give two solutions for (nonzero) $|z|$. The larger value $|z|$ solution may not even correspond to a larger coupling G , as shown in Fig. 5. Also, a G value smaller than the $|z| = 0$ solution is typical. Another illustration of the same coupling value issue is given in Fig. 6 in which we show G versus s plots with two intersecting point, particularly including one with $s = 0$. Such nonzero $|z|$ with $s = 0$ solutions are not available for t less than about .17. For the latter case, the G versus s plots give a single intersecting point. In Fig. 7, we show comparisons of the intersecting point solutions at the same t . We have again solution with larger values of the parameter, s and $|z|$, for the masses generated corresponding again to smaller coupling G . Recall the standard, obviously physical sensible, solution features of the NJL-type model which our $|z| = 0$ solutions shown above bears, is that nontrivial symmetry breaking mass solution exists for large enough coupling beyond a minimal critical value and increases with the coupling. The $|z| \neq 0$ ‘solutions’ behaves, however, in ways difficult to understand. A more careful inspection of the various plots shows that the G_- curve in particular has strange singularities. In fact, each intersecting point ‘solution’ corresponds to a pair of s and $|z|$ values with the G_- curve either diverging at a smaller s or at a smaller $|z|$ value. It sounds like in order to ‘get’ to

that ‘solution’, one has to bring the coupling value all the way to positive or negative infinity and back. However, it should be noted that nonzero $\tilde{\eta}(=|z|\sqrt{t})$ increases the mass of one of the smaller mode but decreases that of the other one [*cf.* Eq.(31)]. It is not so trivial to consider if larger $\tilde{\eta}$ or $|z|$ should really be considered to be giving a larger supersymmetry breaking effect. Another noteworthy feature is that among solutions of fixed $|z|$ a larger t generally tends to give smaller s or \tilde{m}^2 , and among solutions of fixed s , a larger t generally tends to give larger $|z|$; larger t always tends to increase coupling G required for a solution. Recall that the $|m|$ or t value also suppresses the mass generation in the $|z| = 0$ case, but $|m| = 0$ gives certainly no $|z| \neq 0$ solution.

V. THE GOLDSTINO AND COMPOSITE (SUPER)FIELD DYNAMICS

Some components of the superfield U , which are auxiliary as introduced, develop kinetic terms through wavefunction renormalizations in the effective theory below the cutoff Λ . We trace them here through checking of the relevant loop diagrams, based on the effective Lagrangian in terms of components of Φ_R and couplings all having the Φ wavefunction renormalization from the gap equation result incorporated [*cf.* equations in Appendix A]. The analysis focuses on results at the supersymmetry breaking vacuum solutions, *i.e.* nonzero \tilde{m}^2 with zero or nonzero $\tilde{\eta}$. We only sketch the key results here, leaving some more details in Appendix A.

We start with the two-spinors χ and λ . The chirality conserving part of the self-energy diagrams give rise to kinetic terms. We can see all terms are nonzero in the presence of nonvanishing $\tilde{\eta}$, while the χ - λ kinetic mixing vanishes at $\tilde{\eta} = 0$. Full results are presented in Appendix A. To look at the mass values is complicated. One needs first to take a unitary transformation on the hermitian matrix and kinetic terms to diagonalize it. Denote the eigenvalues by N_{f_1} and N_{f_2} , and the diagonalizing matrix by T . The canonically normalized fermionic modes are given by

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{N_{f_1}}} & 0 \\ 0 & \frac{1}{\sqrt{N_{f_2}}} \end{pmatrix} T \begin{pmatrix} \chi \\ \lambda \end{pmatrix}. \quad (37)$$

Only the mass matrix for the canonically modes can be diagonalized to give the mass eigen-

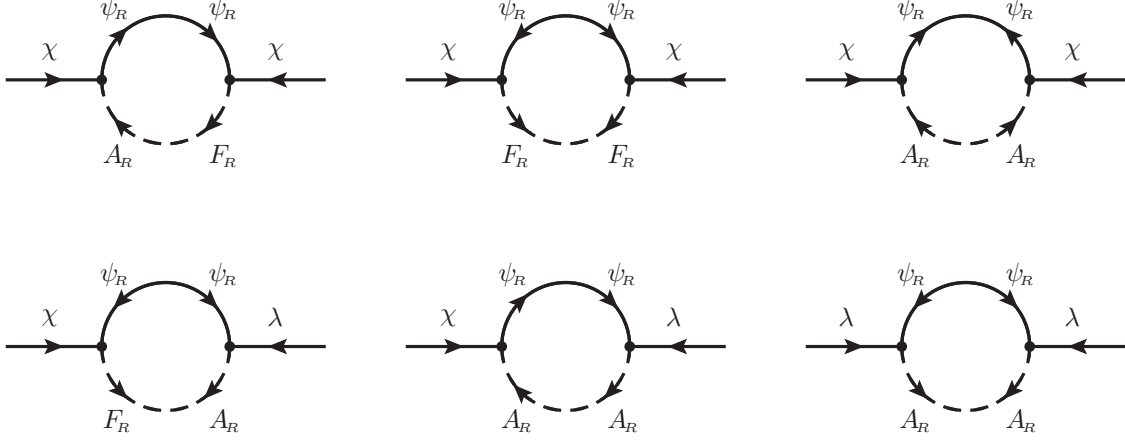


FIG. 8: Diagrams for fermion masses.

values. The mass matrix \mathcal{M}_f for f_1 and f_2 is hence given by

$$\mathcal{M}_f = \begin{pmatrix} \sqrt{N_{f_1}} & 0 \\ 0 & \sqrt{N_{f_2}} \end{pmatrix} T \left(\mathcal{M}_{\chi\lambda} \right) T^T \begin{pmatrix} \sqrt{N_{f_1}} & 0 \\ 0 & \sqrt{N_{f_2}} \end{pmatrix}, \quad (38)$$

where $\mathcal{M}_{\chi\lambda} = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix} + \Omega$, the first part being the tree-level mass while the last is the matrix for chirality-flipping pieces of self-energy diagrams. We have

$$\det \mathcal{M}_f = N_{f_1} N_{f_2} \det \mathcal{M}_{\chi\lambda}. \quad (39)$$

In the case that the matrix of kinetic terms has the full rank, a zero determinant of $\det \mathcal{M}_f$ or equivalently $\det \mathcal{M}_{\chi\lambda}$ shows the existence of a Goldstino, which is to be expected from the supersymmetry breaking. We are here mostly interested only in the kind of qualitative questions here, which saves us from having the deal with the diagonalization of the matrix of kinetic. For the chirality-flipping diagrams (see Fig. 8), dropping the p -dependent parts, we have the mass terms

$$\begin{aligned} \Omega_{\chi\chi} &= -\frac{g^2 \tilde{m}^4}{\tilde{\eta}} |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) + \frac{1}{2\tilde{\eta}} (g^2 I_C + \tilde{m}^2 g^2 I_{N'}) , \\ \Omega_{\chi\lambda} &= 2\mu g^2 \tilde{m}^2 |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) - \mu g^2 I_{N'} , \\ \Omega_{\lambda\lambda} &= -\mu^2 g^2 \tilde{\eta} |m|^2 I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2) , \end{aligned} \quad (40)$$

where $I_{3F}(|m|^2, m_{A_-}^2, m_{A_+}^2)$ is the integral of the product of three Feynman propagators with the mass-squares as specified, and we have expressed the results with the I_C and $I_{N'}$ integrals

of the gap equations [cf. Eqs.(32) and (30)]. Applications of the gap equations kills the term with the then vanishing $(g^2 I_C + \tilde{m}^2 g^2 I_{N'})$ factor and has the $-\mu g^2 I_{N'}$ term canceling the tree-level term in the mass matrix $\mathcal{M}_{\chi\lambda}$ the determinant of which is then exactly zero. Hence, we have established the existence of a Goldstino mode for the supersymmetry breaking solution with $\tilde{\eta} \neq 0$. For the $\tilde{\eta} = 0$ case, only the off-diagonal term is nonzero, which is a result one can see even simply from the $U(1)_R$ symmetry considerations. The latter has been presented in [16], with the result that the tree-level Dirac mass is exactly canceled by Ω -matrix upon application of the corresponding gap equation giving $\mathcal{M}_{\chi\lambda}$ as the zero matrix. Again, we have the Goldstino. Supersymmetry is really a local/spacetime symmetry. The Goldstino would be eaten up by the gravitino which would then be massive.

The spin one vector boson v^μ is an important characteristic of the model. The proper self-energy diagrams (see Fig. 9) for the vector mode give the result

$$\begin{aligned}
\frac{-1}{2}\Sigma_v &= p^2 \frac{\mu^2 g^2}{8} \left\{ 12I_{2F}(|m|^2, |m|^2) - 40|m|^2 I_{3F}(|m|^2, |m|^2, |m|^2) \right. \\
&\quad + 32|m|^2 I_{4F}(|m|^2, |m|^2, |m|^2, |m|^2) + 3I_{2F}(m_{A-}^2, m_{A-}^2) - I_{2F}(m_{A-}^2, m_{A+}^2) \\
&\quad + 3I_{2F}(m_{A+}^2, m_{A+}^2) - 4m_{A-}^2 I_{3F}(m_{A-}^2, m_{A-}^2, m_{A-}^2) - 4m_{A+}^2 I_{3F}(m_{A+}^2, m_{A+}^2, m_{A+}^2) \\
&\quad \left. - \left(m_{A-}^2 + m_{A+}^2 \right) \left[I_{34}(m_{A-}^2, m_{A+}^2) + I_{34}(m_{A+}^2, m_{A-}^2) \right] \right\} \\
&\quad + \frac{\mu^2 g^2}{4} \left\{ 2I_F(|m|^2) + I_F(m_{A-}^2) + I_F(m_{A+}^2) - 4|m|^2 I_F(|m|^2, |m|^2) \right. \\
&\quad \left. - \left(m_{A-}^2 + m_{A+}^2 \right) I_{2F}(m_{A-}^2, m_{A+}^2) \right\} + \dots, \\
\tilde{\eta}=0 &\longrightarrow p^2 \frac{\mu^2 g^2}{8} \left[12I_{2F}(|m|^2, |m|^2) + 5I_{2F}(m_A^2, m_A^2) - 40|m|^2 I_{3F}(|m|^2, |m|^2, |m|^2) \right. \\
&\quad - 20m_A^2 I_{3F}(m_A^2, m_A^2, m_A^2) + 32|m|^2 I_{4F}(|m|^2, |m|^2, |m|^2, |m|^2) \\
&\quad \left. + 16I_{4F}(m_A^2, m_A^2, m_A^2, m_A^2) \right] \\
&\quad + \frac{\mu^2 g^2}{2} \left[I_F(|m|^2) + I_F(m_A^2) - 2|m|^2 I_F(|m|^2, |m|^2) - m_A^2 I_{2F}(m_A^2, m_A^2) \right], \quad (41)
\end{aligned}$$

with I_{nF} denoting the integrals with product of n Feynman propagators. There is also a tree-level mass-square of μ^2 to be added. It sure indicates that we have properly behaved kinetic and mass terms generated (note our metric convention).

The other scalar modes acquire also kinetic and mass terms accordingly. Mode mixings, however, make the result a lot less transparent. Details are given in Appendix A.

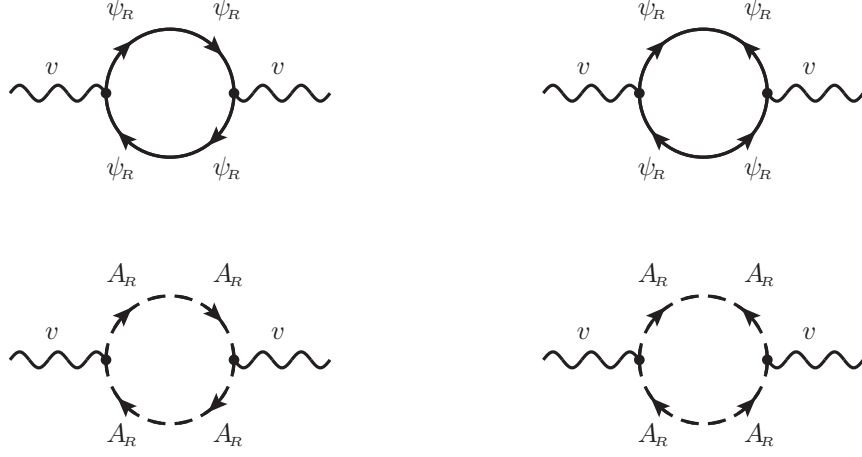


FIG. 9: Proper self-energy diagrams for the spin one composite v^μ .

VI. SOME DISCUSSION ABOUT THE UNCONVENTIONAL FEATURES AND THE VACUUM SOLUTIONS

The model we have here is quite an unconventional one in many aspects, and hence has behavior different from most of the conventional models to the extent that many ‘generic’ features of superfield theory or theory with spontaneous supersymmetry breaking are simply not present. That may make some readers uncomfortable or suspicious. Hence, we want to address the unconventional features directly here as much as we can, in relation to the validity of our main results of the supersymmetry breaking vacuum solution.

First of all, the basic model Lagrangian is unconventional. It has a four-superfield term with like the ‘wrong’ sign and unusual color index contraction, say in comparison to the old SNJL model. The color index contraction gives the NJL-type composite U as an unconventional superfield analysis of which is difficult to find in the literature. We will look at that in more details below. Actually, we are not the first to write down a quartic term in the Kähler potential with a negative sign [23]. To see better its unconventional feature, let us take a look at the component field picture of the model. For simplicity, we again drop the color index from our analysis. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & i\partial_\mu \bar{\psi} \bar{\sigma}^\mu \psi + \partial^\mu A^* \partial_\mu A + F^* F + \left(mAF - \frac{m}{2} \psi\psi + h.c. \right) - \frac{g^2}{2} |2FA - \psi\psi|^2 \\ & + 2g^2 A^* A \partial^\mu A^* \partial_\mu A - 2g^2 i\partial_\mu \bar{\psi} \bar{\sigma}^\mu \psi A^* A - 2g^2 i\bar{\psi} \bar{\sigma}^\mu \psi A \partial_\mu A^* . \end{aligned} \quad (42)$$

From the equation of motion for the auxiliary field F^* , we have

$$F = -\frac{(m^* + g^2\psi\psi)A^*}{1 - 2g^2|A|^2}. \quad (43)$$

The somewhat complicated fractional form of F indicates that the component field Lagrangian with F eliminated would have less than conventional interaction terms. Naively, the scalar potential is given by

$$-V_s = F^*F - 2g^2A^*AF^*F + mAF + m^*A^*F^*. \quad (44)$$

Eliminating F gives, however,

$$V_s = \frac{(|m|^2 - g^4\psi\psi\bar{\psi}\bar{\psi})|A|^2}{1 - 2g^2|A|^2} \quad (45)$$

which formally no longer involves only the scalar. It is suggestive of a bifermion condensate which fits in the general picture of the NJL setting. It is interesting to note that for $m = 0$ the model actually has no pure scalar part in V_s , for any coupling g^2 . On the other hand, if one neglects the fermion field part in the above, the potential looks simple enough, $V_s = \frac{|m|^2|A|^2}{1 - 2g^2|A|^2}$, with a supersymmetric minimum at zero A . For positive g^2 , however, it blows up at $|A| = 1/\sqrt{2}g$. For a perturbative coupling, one expects $1/g$ bigger than the model cut-off scale Λ , hence the potential is well behaved within the cut-off. With strong coupling g^2 , one cannot be so comfortable. In fact, for $|A| > 1/\sqrt{2}g$, the potential goes negative, contradicting our expectation for a supersymmetric model. The analysis so far suggests compatibility with a plausible nonperturbative supersymmetry breaking. In fact, the analysis here illustrates clearly that for strong enough coupling the model has no sensible perturbative vacuum, not even the naive supersymmetric vacuum one may naively expect to work, at least in the $m = 0$ case or for large g^2 .

The nonperturbative NJL-type feature is what gives the model a sensible vacuum. In fact, the model other than being a superfield one has mostly quite conventional NJL-type features at least for the $\tilde{m}^2 \neq 0$ and $\tilde{\eta} = 0$ vacuum. Here below, we mostly address only the latter case as our supersymmetry breaking solution. Let us now take a look at the scalar potential in the presence of the composite U , namely as described by the effective theory Lagrangian, for $m = 0$ at the tree-level. We have to emphasize that the effective theory really comes from the NJL-type composite (super)field thinking consistence of which asks for the potential analysis as performed above in Sec. III. We are looking at the tree-level

potential here only to illustrate further the unconventional features of the model, here as given by the effective theory Lagrangian. The potential has the very unconventional form given by

$$V_{eff}^{tree} = -F_R^* F_R - g C F_R^* F_R - \mu C D - N^* N - \mu g D A_R^* A_R - g N A_R F_R^* - g N^* A_R^* F_R, \quad (46)$$

which has vanishing minima both at the trivial supersymmetric origin of the field space except with arbitrary value for A_R , and at $C = D = 0$ and $|A_R| = 1/g$ with F_R and $N = -g A_R^* F_R$ undetermined. The latter potentially supersymmetry breaking minimum is fully consistent with expressions one may obtain naively for eliminating any of the auxiliary scalars including F . Taking $m = 0$ simplifies the analysis. It is also really the most interesting benchmark case in which there is no input mass parameter at all. Let us emphasize again that the composite superfield U though has a spin one component v^μ is not at all the conventional real scalar superfield of a gauge boson. Before putting in the ‘quark-loop’ correction, it has no kinetic term and the conventional D^2 term is missing. It is sure massive, and does not even have the right couplings to be considered as the gauge boson with broken gauge symmetry.

Eliminating all C , N , and D from V_{eff}^{tree} of course gives back only the V_s potential of the origin Lagrangian. Those conditions are really from the composite condition of $U = -\frac{g}{\mu} \bar{\Phi}_R \Phi_R$ the NJL wisdom of nontrivial two field condensates says exactly that they should not be applied to the VEVs. A simple conclusion here is that while our gap equations above allows a supersymmetry preserving solution, the conventional wisdom of that being the preferred vacuum may not apply. At least there is no indications that the supersymmetry breaking vacuum is less stable. In fact, the supersymmetric solution means there is no two-field condensate of Φ except possibly what contributes to $\langle C \rangle$. The latter is just a wavefunction renormalization factor for Φ which is already absorbed in V_{eff}^{tree} and gives the same form for V_s in terms of renormalized quantities. After all, without symmetry breaking two-field condensate is like what one expect with weak g^2 coupling, there should not be any composite field degree of freedom and the perturbative tree-level V_s should be expected to give the correct qualitative feature of the solution. However, the latter looks unstable or sick for strong enough coupling.

Another point to note is that the gap equation analysis for an NJL-type model always seems to admit the symmetry preserving solution [11, 13, 14]. However, we can see that the

symmetry preserving solution looks applicable only to the case of weak g^2 , or subcritical G coupling. The derivations of the gap equation(s) have taken no constraint on the coupling. Nontrivial solution is not possible with a subcritical coupling. Hence, admitting the trivial solution is really a like consistence condition. Now if one assumes the composite formation and looks at the effective theory, the following argument says the trivial, symmetry preserving solution should not be considered valid. At least in principle, we can take the effective theory with the ‘quark-loop’ contributions included without putting in any symmetry breaking VEVs. That has been done for the NJL and the SNJL models, for example, to retrieve effective potential for the SM and the (M)SSM [1, 24]. Those scalar potentials do not admit the symmetry preserving vacuum solutions. We want to emphasize again that the effective theory picture should be taken as valid only with the strong coupling. Note further that the gap equations as obtained from the tadpole analysis of the potential for the effective theory formally gives only vanishing first derivative conditions, hence turning points rather than sure minimum. For the SM or the (M)SSM, the zero VEV symmetry preserving point for the Higgs potential is sure a turning point, and indeed a local maximum. In that light, the nature of the supersymmetry preserving point as a vacuum solution may be questionable at large coupling. That thinking is in consistent with the V_s analysis.

From all the above, we conclude that there is no clear indication of the supersymmetry breaking vacua being metastable or not stable enough. It actually may be the preferred solution for the case of strong coupling. It will of course be very nice if what we argue for here above can be rigorous demonstrated to be the case with some further analysis, probably beyond the large N approximation as inherent in the basic NJL-type analysis here. A related and important issue is the relative stability of the different supersymmetry breaking vacua. While it is difficult to check if the same strong coupling value admits more than one such vacua, it is seems certain to be the case for the coupling values that admit $\tilde{\eta} \neq 0$ solution(s), as the existence of $\tilde{\eta} = 0$ and $\tilde{m}^2 \neq 0$ solution is generic once the coupling is beyond the critical value. The problem is for the nonzero input m case only though.

Another important aspect about our model that some may feel suspicious is its being able to avoid the vanishing supertrace condition for the mass-squares of the component fields, which is in general difficult. However, that the condition was established only for specific models of supersymmetry breaking rather than as a generic result [25–27], though the class of models include most of the better known ones. It sure does not work for the case of

an anomalous Fayet-Iliopoulos D -term for example. There is no good reason to assume any conclusion on the issue about the case of our model. If there is at all any somewhat similar structure in our model to those more conventional ones in terms of supersymmetry breaking, it may be with the Fayet-Iliopoulos case with a potential sort of linear in the D -term. And we sure have no anomaly issue. Our analysis clearly indicates generation of only soft supersymmetry breaking masses for the scalar component A and not the fermion ψ . There cannot be any doubt about that. In fact, the $m = 0$ case with $\tilde{m}^2 \neq 0$ and $\tilde{\eta} = 0$ solution is even more illustrative. There are clearly masses for A and the composite spin-one v^μ generated, while no mass term for any fermionic mode at all. Avoiding the vanishing supertrace condition of course is the central feature that we are after, with particularly interesting application phenomenologically.

VII. REMARKS AND CONCLUSIONS

Dynamical supersymmetry breaking is an interesting and important topic [28]. Our new simple model with a single chiral superfield (multiplet) should be a great addition to the latter. The fact that the model has as its direct consequence the generation of soft supersymmetry breaking masses is specially interesting in view of the requirement of soft supersymmetry breaking in any low energy phenomenological application of supersymmetry as in a SSM. We want to emphasize that our key interest is really the case with an $SU(N)$ multiplet and no input mass parameter, *i.e.* $m = 0$. For the case, the analysis can simply be considered one with the color index hidden so long as we put back the color factor N in the relevant loop diagrams, basically has the g^2 factor in all those results including the gap equations to be replaced by $g^2 N$. We may also take the superfield as like one of the chiral matter superfield multiplets in the SSM. Together with our earlier HSNJL model [9, 13], a simple SSM with all (super)symmetry breaking and mass parameters generated dynamically is easily in sight, though it remains to see if a model with only the SSM superfield spectrum minus the Higgs supermultiplets can be a consistent model theoretically and phenomenologically. Even if the answer to the latter question is a no, it looks like there is at least enough room to have a model with like a single extra chiral superfield to produce the supersymmetry breaking. We consider a model of such kind quite compelling as an alternative to the full models of the SSM in the literature having the extra sectors.

The current study is a big step in the direction, to which we sure love to further our investigations. The model mechanism of course may also be applied to other model building works, for example in addressing the (S)SM flavor structure questions [29].

We take only the case of a simple singlet composite of $U \sim \Phi_a^\dagger \Phi^a$ here. A somewhat more complicated case as studied in the case of (non-supersymmetric) NJL-type composite of spin one field [30] would have the composite in the adjoint representation. Similar but superfield version of four-superfield interactions may be considered though not in relation to pure soft supersymmetry breaking. It is also possible to have a model in which the composite superfield U behaves like a massive gauge field supermultiplet [31], much in parallel with the non-supersymmetric models of Ref.[30]. Note again that our current model does not have the right coupling for the spin one field v_μ to behave like a gauge boson at all. There is no $A^* v^\mu v_\mu A$ term in the Lagrangian [*cf.* Eq.(19) and Eq.(A1)], or no $\bar{\Phi} U^2 \Phi$ term Eq.(17) in the superfield picture. It is possible to think about the electroweak gauge bosons as such composites. However, we echo the author of Ref.[30] against advocating the kind of scenario.

Finally, we emphasize that with the modern effective (field) theory perspective, it is the most natural thing to consider any theory as an effective description of Nature only within a limited domain/scale. Physics is arguably only about effective theories, as any theory can only be verified experimentally up to a finite scale and there may always be a cut-off beyond that. Having a cutoff scale with the so-called nonrenormalizable higher dimensional operators is hence in no sense an undesirable feature. Model content not admitting any other parameter with mass dimension in the Lagrangian would be very natural. Dynamical mass generation with symmetry breaking is then necessary to give the usual kind of low energy phenomenology such as the Standard Model one. That is actually the key motivation behind our line of work on the subject matter.

Appendix A: Some more technical details of the model calculations

Starting with the effective Lagrangian of Eq.(19) with c , d , and n denoting VEVs of the (original) scalars C , D , and N , we have, in terms of the renormalized components $A_R = \sqrt{Z}A$, $\psi_R = \sqrt{Z}\psi$, and $F_R = \sqrt{Z}F$ of $\Phi_R = \sqrt{Z}\Phi$ with the common (supersymmetric) wavefunction renormalization factor $Z = 1 + g_o c$, the quadratic part of the Lagrangian is

given by

$$\begin{aligned} \mathcal{L}_{eff}^{(2)} = & A_R^* \square A_R + i(\partial_\mu \bar{\psi}_R) \bar{\sigma}^\mu \psi_R + F_R^* F_R + \frac{m}{2} (2A_R F_R - \psi_R \psi_R) + \frac{m^*}{2} (2A_R^* F_R^* - \bar{\psi}_R \bar{\psi}_R) \\ & + \mu C D - \mu \chi \lambda - \mu \bar{\chi} \bar{\lambda} + N \bar{N} - \frac{\mu^2}{2} v^\nu v_\nu + \mu g d A_R^* A_R + g n A_R F_R^* + g n^* A_R^* F_R, \quad (A1) \end{aligned}$$

in which we have the renormalized mass and coupling $m = \frac{m_0}{Z}$ and $g = \frac{g_0}{Z}$. Here the scalars C , N , and D are the physical ones with VEVs already pull out, though we do not distinguish them from the original ones with VEVs explicitly in notation. One can easily obtain the following propagator expressions :

$$\begin{aligned} \langle T(A_R A_R^*) \rangle &= \frac{-i(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(A_R A_R) \rangle &= \frac{2i g n^* m^*}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(F_R F_R^*) \rangle &= \frac{i(p^2 - \mu g d)(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(F_R F_R) \rangle &= \frac{-2i g n m^* (p^2 - \mu g d)}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(A_R F_R) \rangle &= \frac{i m^* (p^2 + |m|^2 - g^2 |n|^2 - \mu g d)}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(A_R F_R^*) \rangle &= \frac{i g n^* (p^2 - |m|^2 + g^2 |n|^2 - \mu g d)}{(p^2 + |m|^2 + g^2 |n|^2 - \mu g d)^2 - 4g^2 |n|^2 |m|^2}, \\ \langle T(\psi_{R\alpha} \bar{\psi}_{R\dot{\beta}}) \rangle &= \frac{-i p_\mu \sigma_{\alpha\dot{\beta}}^\mu}{p^2 + |m|^2}, \\ \langle T(\psi_{R\alpha} \psi_R^\beta) \rangle &= \frac{-i m^* \delta_\alpha^\beta}{p^2 + |m|^2}. \quad (A2) \end{aligned}$$

Note that $-\mu g d$ and $-g n$ here correspond to the (renormalized) soft mass terms \tilde{m}^2 and $\tilde{\eta}$. The propagator expressions can be matched to that of the superfield Φ in Eqn.(12).

The remaining, interaction, terms in the effective Lagrangian read

$$\begin{aligned} \mathcal{L}_{eff}^{int} = & g C [A_R^* \square A_R + i(\partial_\mu \bar{\psi}_R) \bar{\sigma}^\mu \psi_R + F_R^* F_R] - \mu g \psi_R \lambda A_R^* - \mu g \bar{\psi}_R \bar{\lambda} A_R + \mu g D A_R^* A_R \\ & - i \frac{g}{2} \bar{\psi}_R \bar{\sigma}^\mu \chi \partial_\mu A_R + i \frac{g}{2} (\partial_\mu \bar{\psi}_R) \bar{\sigma}^\mu \chi A_R - g \chi \psi_R F_R^* + g N A_R F_R^* \\ & + i \frac{g}{2} \bar{\chi} \bar{\sigma}^\mu \psi_R \partial_\mu A_R^* - i \frac{g}{2} A_R^* \bar{\chi} \bar{\sigma}^\mu \partial_\mu \psi_R - g \bar{\chi} \bar{\psi}_R F_R + g N^* A_R^* F_R \\ & - \frac{\mu g}{\sqrt{2}} \eta^{\mu\nu} v_\mu i A_R^* \partial_\nu A_R + \frac{\mu g}{\sqrt{2}} \eta^{\mu\nu} v_\mu i (\partial_\nu A_R^*) A_R - \frac{\mu g}{\sqrt{2}} \eta^{\mu\nu} v_\mu \bar{\psi}_R \bar{\sigma}_\nu \psi_R. \quad (A3) \end{aligned}$$

Note that the above gives essentially all parts of the Lagrangian, apart from a constant. The linear terms are canceled at the physical vacuum with consistent c , n , d solutions discussed in the main text.

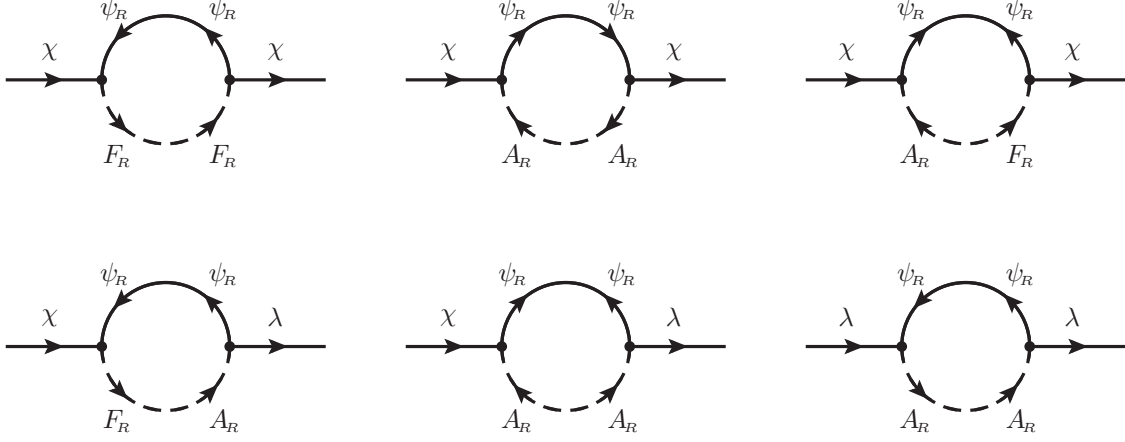


FIG. 10: Diagrams for the generation of kinetic terms for the fermionic modes.

In the following, we present some details of the ‘quark-loop’ contribution to the two-point functions for the various components of the composite superfield U at the supersymmetry breaking vacuum solutions, as discussed in Sec. V. Though we argue in the text that $\tilde{\eta} \neq 0$ solution does not look acceptable, we present fully generic results for completion. The results may offer more insight into the problem.

The two-point functions for fermion kinetic terms are given by the diagrams in Fig. 10, with the $ip \cdot \bar{\sigma} \Xi$ results given as ⁶

$$\begin{aligned}
\Xi_{\chi\chi} = & \frac{g^2}{4} \left[|m| (|m| - |\tilde{\eta}|) I_{2F}(|m|^2, m_{A_-}^2) + |m| (|m| + |\tilde{\eta}|) I_{2F}(|m|^2, m_{A_+}^2) \right. \\
& + 2I_F(m_{A_-}^2) - 2(2\tilde{m}^2 + 3|m|^2 - 2|\tilde{\eta}||m|) I_{2F}(m_{A_-}^2, m_{A_-}^2) \\
& + 2I_F(m_{A_+}^2) - 2(2\tilde{m}^2 + 3|m|^2 + 2|\tilde{\eta}||m|) I_{2F}(m_{A_+}^2, m_{A_+}^2) \\
& - 2|m|^2 (m_{A_-}^2 - 2\tilde{m}^2 - 3|m|^2 + 2|\tilde{\eta}||m|) I_{3F}(|m|^2, m_{A_-}^2, m_{A_-}^2) \\
& \left. - 2|m|^2 (m_{A_+}^2 - 2\tilde{m}^2 - 3|m|^2 - 2|\tilde{\eta}||m|) I_{3F}(|m|^2, m_{A_+}^2, m_{A_+}^2) \right] + \dots, \\
\tilde{\eta}=0 \longrightarrow & \frac{g^2}{2} \left[2I_F(m_A^2) + |m|^2 I_{2F}(|m|^2, m_A^2) - 2(2\tilde{m}^2 + 3|m|^2) I_{2F}(m_A^2, m_A^2) \right. \\
& \left. + 2|m|^2 (\tilde{m}^2 + 2|m|^2) I_{3F}(|m|^2, m_A^2, m_A^2) \right], \tag{A4}
\end{aligned}$$

⁶ A good reference for the technical aspects of the calculations by Ref.[32] notation of which we more or less follow, except that our background notation/convention is based on Wess and Bagger [17].

$$\begin{aligned}
\Xi_{\chi\lambda} = & \mu g^2 \tilde{\eta}^* \left[\left(1 - \frac{|m|}{4|\tilde{\eta}|}\right) I_{2F}(|m|^2, m_{A-}^2) + \left(1 + \frac{|m|}{4|\tilde{\eta}|}\right) I_{2F}(|m|^2, m_{A+}^2) \right. \\
& \left. - m_{A-}^2 I_{3F}(|m|^2, m_{A-}^2, m_{A-}^2) - m_{A+}^2 I_{3F}(|m|^2, m_{A+}^2, m_{A+}^2) \right] + \dots, \\
\tilde{\eta}=0 \longrightarrow & 0.
\end{aligned} \tag{A5}$$

$$\begin{aligned}
\Xi_{\lambda\lambda} = & \frac{\mu^2 g^2}{2} \left[I_{2F}(m_{A-}^2, m_{A-}^2) + I_{2F}(m_{A+}^2, m_{A+}^2) \right. \\
& \left. - |m|^2 I_{3F}(|m|^2, m_{A-}^2, m_{A-}^2) - |m|^2 I_{3F}(|m|^2, m_{A+}^2, m_{A+}^2) \right] + \dots, \\
\tilde{\eta}=0 \longrightarrow & 2\mu^2 g^2 \left[I_{2F}(m_A^2, m_A^2) - |m|^2 I_{3F}(|m|^2, m_A^2, m_A^2) \right],
\end{aligned} \tag{A6}$$

where I_{nF} denote integrals each of a product of n Feynman propagators with the mass-square parameters as given. We have given besides the general result also the simplified expression at the $\tilde{\eta} = 0$ limit. Recall

$$m_{A\mp}^2 = \tilde{m}^2 + (|m| \mp |\tilde{\eta}|)^2$$

and at the limit $\tilde{\eta} = 0$, we have used $m_A^2 \equiv \tilde{m}^2 + |m|^2 = m_{A-}^2 = m_{A+}^2$. The vanishing kinetic mixing between χ and λ can also be easily seen from $U(1)_R$ symmetry considerations.

For the scalars, results for the various proper self-energy diagrams are very tedious. Apart from I_{nF} , we further introduce

$$I_{34}(m_a^2, m_b^2) \equiv 3I_{3F}(m_a^2, m_b^2, m_b^2) - 4m_b^2 I_{4F}(m_a^2, m_b^2, m_b^2, m_b^2), \tag{A7}$$

to present the results. Again, we give the general result and the $\tilde{\eta} = 0$ limit, and the



FIG. 11: Proper self-energy diagrams for the DD term.

diagrams are given in the Figs (11) to (17). We have:

$$\begin{aligned}
\frac{-1}{2}\Sigma_{DD} &= p^2 \frac{\mu^2 g^2}{8} [I_{34}(m_{A_-}^2, m_{A_-}^2) + I_{34}(m_{A_+}^2, m_{A_+}^2)] \\
&\quad + \frac{\mu^2 g^2}{8} [I_{2F}(m_{A_-}^2, m_{A_-}^2) + I_{2F}(m_{A_+}^2, m_{A_+}^2)] + \dots, \\
\tilde{\eta}=0 &\longrightarrow p^2 \frac{\mu^2 g^2}{4} I_{34}(m_A^2, m_A^2) + \frac{\mu^2 g^2}{4} I_{2F}(m_A^2, m_A^2), \tag{A8}
\end{aligned}$$

$$\begin{aligned}
\frac{-1}{2}\Sigma_{CD} &= p^2 \frac{\mu g^2}{4} \left\{ \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{34}(m_{A_-}^2, m_{A_-}^2) + \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{34}(m_{A_+}^2, m_{A_+}^2) \right. \\
&\quad + \frac{\mu g^2}{4} \left\{ \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_-}^2, m_{A_-}^2) + \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_+}^2, m_{A_+}^2) \right. \\
&\quad \left. \left. - I_F(m_{A_-}^2) - I_F(m_{A_+}^2) \right\} + \dots, \right. \\
\tilde{\eta}=0 &\longrightarrow p^2 \frac{\mu g^2}{2} (m_A^2 + |m|^2) I_{34}(m_A^2, m_A^2) \\
&\quad + \frac{\mu g^2}{2} \left[(m_A^2 + |m|^2) I_{2F}(m_A^2, m_A^2) - I_F(m_A^2) \right], \tag{A9}
\end{aligned}$$

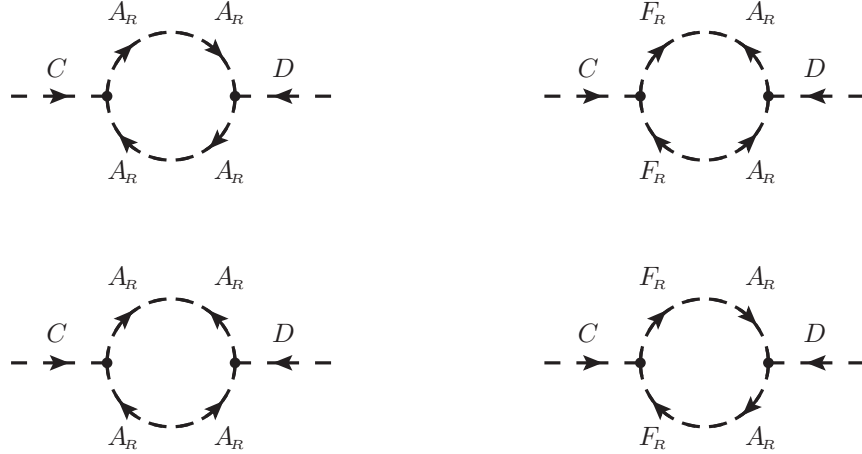


FIG. 12: Proper self-energy diagrams for the CD term.

$$\begin{aligned}
\frac{-1}{2}\Sigma_{cc} = & p^2 \frac{g^2}{8} \left\{ \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right]^2 I_{34}(m_{A_-}^2, m_{A_-}^2) - 2|\tilde{\eta}||m|m_{A_+}^2 I_{34}(m_{A_-}^2, m_{A_+}^2) \right. \\
& + 2|\tilde{\eta}||m|m_{A_-}^2 I_{34}(m_{A_+}^2, m_{A_-}^2) + \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right]^2 I_{34}(m_{A_+}^2, m_{A_+}^2) \\
& - 3 \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_-}^2, m_{A_-}^2) - 3 \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_+}^2, m_{A_+}^2) \\
& + 4m_{A_-}^2 \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{3F}(m_{A_-}^2, m_{A_-}^2, m_{A_-}^2) \\
& \left. + 4m_{A_+}^2 \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{3F}(m_{A_+}^2, m_{A_+}^2, m_{A_+}^2) \right\} \\
& + \frac{g^2}{16} \left\{ 16 - 8|m|^2 I_F(|m|^2) + \left[m_{A_+}^2 - 5m_{A_-}^2 - 8(|m| - |\tilde{\eta}|)^2 \right] I_F(m_{A_-}^2) \right. \\
& - \left[5m_{A_+}^2 - m_{A_-}^2 + 8(|m| + |\tilde{\eta}|)^2 \right] I_F(m_{A_+}^2) \\
& + \left[2m_{A_-}^2 m_{A_+}^2 - m_{A_+}^2 m_{A_+}^2 - m_{A_-}^2 m_{A_-}^2 \right] I_{2F}(m_{A_-}^2, m_{A_+}^2) \\
& + 2 \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right]^2 I_{2F}(m_{A_-}^2, m_{A_-}^2) \\
& \left. + 2 \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right]^2 I_{2F}(m_{A_+}^2, m_{A_+}^2) \right\} + \dots, \\
\tilde{\eta}=0 \longrightarrow & p^2 \frac{g^2}{4} \left[4m_A^2 (m_A^2 + |m|^2) I_{3F}(m_A^2, m_A^2, m_A^2) + (m_A^2 + |m|^2)^2 I_{34}(m_A^2, m_A^2) \right. \\
& \left. - 3 (m_A^2 + |m|^2) I_{2F}(m_A^2, m_A^2) \right] \\
& + \frac{g^2}{4} \left[4 - 2|m|^2 I_F(|m|^2) - 2 (m_A^2 + 2|m|^2) I_F(m_A^2) \right. \\
& \left. + (m_A^2 + |m|^2)^2 I_{2F}(m_A^2, m_A^2) \right], \tag{A10}
\end{aligned}$$

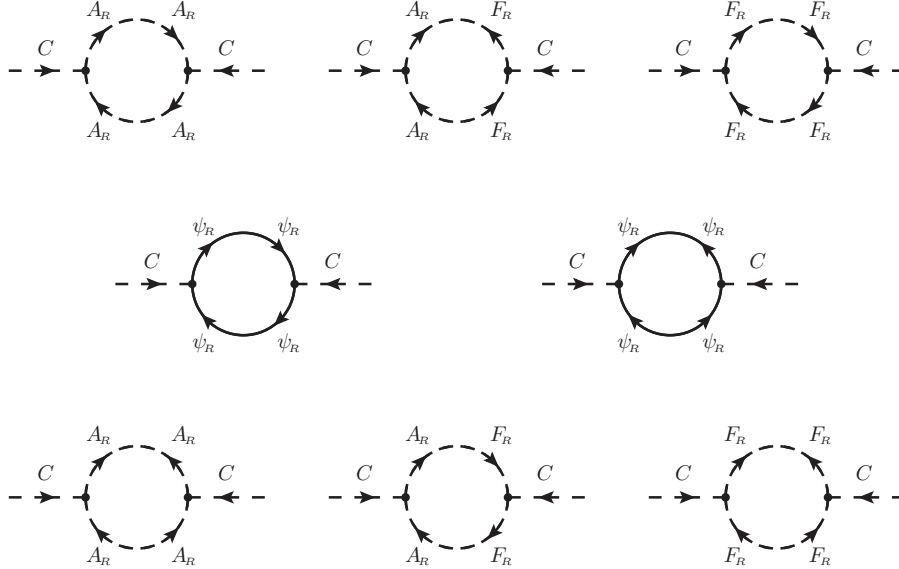


FIG. 13: Proper self-energy diagrams for the CC term.



FIG. 14: Proper self-energy diagrams for the NN^* term.



FIG. 15: Proper self-energy diagrams for the NN term.

$$\begin{aligned}
-\Sigma_{NN^*} &= p^2 \frac{g^2}{2} \left\{ (|m| - |\tilde{\eta}|)^2 I_{34}(m_{A_-}^2, m_{A_-}^2) + (|m| + |\tilde{\eta}|)^2 I_{34}(m_{A_+}^2, m_{A_+}^2) \right. \\
&\quad \left. + |m| (|m| + |\tilde{\eta}|) I_{34}(m_{A_-}^2, m_{A_+}^2) + |m| (|m| - |\tilde{\eta}|) I_{34}(m_{A_+}^2, m_{A_-}^2) \right. \\
&\quad \left. + \frac{g^2}{2} \left\{ (|m| - |\tilde{\eta}|)^2 I_{2F}(m_{A_-}^2, m_{A_-}^2) + (|m| + |\tilde{\eta}|)^2 I_{2F}(m_{A_+}^2, m_{A_+}^2) \right. \right. \\
&\quad \left. \left. + 2|m|^2 I_{2F}(m_{A_-}^2, m_{A_+}^2) - I_F(m_{A_-}^2) - I_F(m_{A_+}^2) \right\} + \dots \right\}, \\
\tilde{\eta}=0 &\longrightarrow p^2 2g^2 |m|^2 I_{34}(m_A^2, m_A^2) + g^2 [2|m|^2 I_{2F}(m_A^2, m_A^2) - I_F(m_A^2)] . \quad (A11)
\end{aligned}$$

There are more mixing terms which vanish with $\tilde{\eta}$, as follows:

$$\begin{aligned}
-\Sigma_{NN} &= p^2 \frac{g^2}{4} \frac{\tilde{\eta}^2}{|\tilde{\eta}|^2} \left[(|m| - |\tilde{\eta}|)^2 I_{34}(m_{A_-}^2, m_{A_-}^2) + (|m| + |\tilde{\eta}|)^2 I_{34}(m_{A_+}^2, m_{A_+}^2) \right. \\
&\quad \left. - |m| (|m| - |\tilde{\eta}|) I_{34}(m_{A_-}^2, m_{A_+}^2) - |m| (|m| + |\tilde{\eta}|) I_{34}(m_{A_+}^2, m_{A_-}^2) \right] \\
&\quad + \frac{g^2}{4} \frac{\tilde{\eta}^2}{|\tilde{\eta}|^2} \left[(|m| - |\tilde{\eta}|)^2 I_{2F}(m_{A_-}^2, m_{A_-}^2) + (|m| + |\tilde{\eta}|)^2 I_{2F}(m_{A_+}^2, m_{A_+}^2) \right. \\
&\quad \left. - 2|m|^2 I_{2F}(m_{A_-}^2, m_{A_+}^2) \right], \quad (A12)
\end{aligned}$$

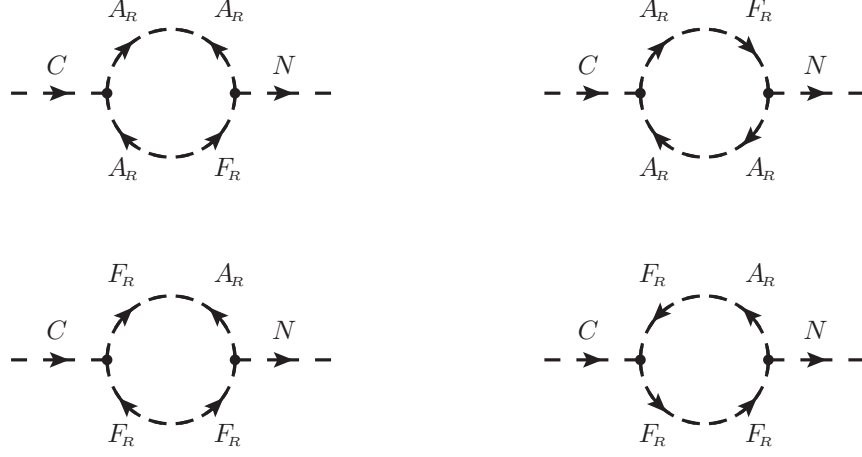


FIG. 16: Proper self-energy diagrams for the CN^* term.



FIG. 17: Proper self-energy diagrams for the DN^* term.

$$\begin{aligned}
-\Sigma_{CN^*} = & p^2 \frac{g^2}{2} \frac{\tilde{\eta}}{|\tilde{\eta}|} \left\{ (|m| + |\tilde{\eta}|) \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{34}(m_{A_+}^2, m_{A_+}^2) - m_{A_+}^2 |m| I_{34}(m_{A_-}^2, m_{A_+}^2) \right. \\
& \left. + m_{A_-}^2 |m| I_{34}(m_{A_+}^2, m_{A_-}^2) - (|m| - |\tilde{\eta}|) \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{34}(m_{A_-}^2, m_{A_-}^2) \right\} \\
& + \frac{g^2}{2} \frac{\tilde{\eta}}{|\tilde{\eta}|} \left\{ (3|m| - 2|\tilde{\eta}|) I_F(m_{A_-}^2) - (3|m| + 2|\tilde{\eta}|) I_F(m_{A_+}^2) - 4|m|^2 |\tilde{\eta}| I_{2F}(m_{A_-}^2, m_{A_+}^2) \right. \\
& - (|m| - |\tilde{\eta}|) \left[m_{A_-}^2 + (|m| - |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_-}^2, m_{A_-}^2) \\
& \left. + (|m| + |\tilde{\eta}|) \left[m_{A_+}^2 + (|m| + |\tilde{\eta}|)^2 \right] I_{2F}(m_{A_+}^2, m_{A_+}^2) \right\}, \quad (A13)
\end{aligned}$$

$$\begin{aligned}
-\Sigma_{DN^*} = & p^2 \frac{\mu g^2}{2} \frac{\tilde{\eta}}{|\tilde{\eta}|} \left[(|m| + |\tilde{\eta}|) I_{34}(m_{A_+}^2, m_{A_+}^2) - (|m| - |\tilde{\eta}|) I_{34}(m_{A_-}^2, m_{A_-}^2) \right. \\
& \left. - |m| I_{34}(m_{A_-}^2, m_{A_+}^2) + |m| I_{34}(m_{A_+}^2, m_{A_-}^2) \right] \\
& + \frac{\mu g^2}{2} \frac{\tilde{\eta}}{|\tilde{\eta}|} \left\{ (|m| + |\tilde{\eta}|) I_{2F}(m_{A_+}^2, m_{A_+}^2) - (|m| - |\tilde{\eta}|) I_{2F}(m_{A_-}^2, m_{A_-}^2) \right\}, \quad (A14)
\end{aligned}$$

with also the complex conjugates for the last three, *i.e.* $-\Sigma_{N^*N^*}$, $-\Sigma_{CN}$, and $-\Sigma_{DN}$.

It is somewhat of a surprise that all the scalar actually becomes kinetic, including D and N . The latter are introduced as auxiliary components of mass dimension two. One should hence consider $\frac{D}{\mu}$ and $\frac{N}{\mu}$ instead. For general case, the complex $\frac{N}{\mu}$ has to be expanded into the real components first. One has then to diagonalize the kinetic term matrix for all the real scalars to find the proper wavefunction renormalization factors for the canonical modes, and subsequently diagonalize the mass-square matrix, with the tree-level terms included, of the latter for the eigenvalues.

Appendix B: Propagator expressions for the most general case

We give here the superfield propagator expressions for the most general case, *i.e.* all soft supersymmetry breaking parameters are included. The propagator(s) used in our model above is the case with the soft mass term $-\frac{1}{2}\eta\theta^2\Phi^2$ in the superpotential vanishing. The expressions have, apparently, not been explicitly given before, and may be useful in some future studies.

The free-field Lagrangian for a single chiral superfield $\Phi = A + \sqrt{2}\psi\theta + F\theta^2$ admitting all supersymmetric and (soft) supersymmetry breaking mass parameter can be written as

$$\mathcal{L}_o = \int d^4\theta \bar{\Phi}\Phi(1 - \tilde{\eta}\theta^2 - \tilde{\eta}^*\bar{\theta}^2 - \tilde{m}^2\theta^2\bar{\theta}^2) + \left[\int d^2\theta \frac{1}{2}(m - \eta\theta^2)\Phi^2\delta^2(\bar{\theta}) + h.c. \right]. \quad (\text{B1})$$

Again, we allow a complex m . Soft supersymmetry breaking parameters $\tilde{\eta}$ and η are also complex while the most familiar soft mass \tilde{m}^2 is real. The superfield propagators are given by

$$\begin{aligned} \langle T(\Phi(1)\Phi^\dagger(2)) \rangle &= \frac{-i}{p^2 + |m|^2} \delta_{12}^4 - \frac{i[\tilde{\eta}(Q - 2|m|^2) + m^*\eta]}{Q^2 - |\eta - 2m\tilde{\eta}|^2} \theta_1^2 \delta_{12}^4 - \frac{i[\tilde{\eta}^*(Q - 2|m|^2) + m\eta^*]}{Q^2 - |\eta - 2m\tilde{\eta}|^2} \bar{\theta}_1^2 \delta_{12}^4 \\ &+ i \frac{(-p^2|\tilde{\eta}|^2 + \tilde{m}^2|m|^2)Q + 4p^2|m|^2|\tilde{\eta}|^2 - (p^2 - |m|^2)(m^*\eta\tilde{\eta}^* + m\eta^*\tilde{\eta}) - |m|^2|\eta|^2}{(p^2 + |m|^2)(Q^2 - |\eta - 2m\tilde{\eta}|^2)} \theta_1^2 \bar{\theta}_1^2 \delta_{12}^4 \\ &+ i \frac{(\tilde{m}^2 + |\tilde{\eta}|^2)Q - |\eta - 2m\tilde{\eta}|^2}{(p^2 + |m|^2)(Q^2 - |\eta - 2m\tilde{\eta}|^2)} \left[\frac{D_1^2 \theta_1^2 \bar{\theta}_1^2 \bar{D}_1^2}{16} \right] \delta_{12}^4, \end{aligned} \quad (\text{B2})$$

and

$$\begin{aligned}
\langle T(\Phi(1)\Phi(2)) \rangle &= \frac{i m^*}{p^2(p^2 + |m|^2)} \frac{D_1^2}{4} \delta_{12}^4 - \frac{i(\eta^* - 2m^* \tilde{\eta}^*)}{Q^2 - |\eta - 2m\tilde{\eta}|^2} \frac{D_1^2 \bar{\theta}_1^2}{4} \delta_{12}^4 \\
&+ i \frac{2m^* \tilde{\eta}(p^2 + \tilde{m}^2) + m^{*2} \eta + \eta^* \tilde{\eta}^2}{Q^2 - |\eta - 2m\tilde{\eta}|^2} \frac{D_1^2 \theta_1^2}{4p^2} \delta_{12}^4 \\
&+ i \frac{m^*[(\tilde{m}^2 + |\tilde{\eta}|^2)Q - |\eta - 2m\tilde{\eta}|^2] - \tilde{\eta}(\eta^* - 2m^* \tilde{\eta}^*)(p^2 + |m|^2)}{(p^2 + |m|^2)(Q^2 - |\eta - 2m\tilde{\eta}|^2)} \left[\frac{D_1^2 \theta_1^2 \bar{\theta}_1^2}{4} + \frac{\bar{\theta}_1^2 \theta_1^2 D_1^2}{4} \right] \delta_{12}^4.
\end{aligned} \tag{B3}$$

where $Q = p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2$. The corresponding component field propagators are given by

$$\begin{aligned}
\langle T(A A^*) \rangle &= \frac{-i(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(A A) \rangle &= \frac{i(\eta^* - 2m^* \tilde{\eta}^*)}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(F F^*) \rangle &= \frac{i[(p^2 + \tilde{m}^2)(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) - |\eta - m\tilde{\eta}|^2 + |m\tilde{\eta}|^2]}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(F F) \rangle &= \frac{i[2m^* \tilde{\eta}(p^2 + \tilde{m}^2) + m^{*2} \eta + \eta^* \tilde{\eta}^2]}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(A F) \rangle &= \frac{i[m^*(p^2 + |m|^2 + \tilde{m}^2 - |\tilde{\eta}|^2) + \eta^* \tilde{\eta}]}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(A F^*) \rangle &= \frac{-i[\tilde{\eta}^*(p^2 - |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2) + m\eta^*]}{(p^2 + |m|^2 + \tilde{m}^2 + |\tilde{\eta}|^2)^2 - |\eta - 2m\tilde{\eta}|^2}, \\
\langle T(\psi_{R\alpha} \bar{\psi}_{R\dot{\beta}}) \rangle &= \frac{-ip_\mu \sigma_{\alpha\dot{\beta}}^\mu}{p^2 + |m|^2}, \\
\langle T(\psi_{R\alpha} \psi_R^\beta) \rangle &= \frac{-im^* \delta_\alpha^\beta}{p^2 + |m|^2}.
\end{aligned} \tag{B4}$$

Note that the Lagrangian without all the masses has a $U(1)$ and a $U(1)_R$ symmetry to which Φ carries both charges (of 1). $U(1)_R$ charges for the components A and F are 1 and -1, with ψ neutral. We can assign corresponding charges to the mass parameters and use them to help trace and check the role of the parameters in the component field propagators and the corresponding terms of the superfield propagators.

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- [1] For a review, see G. Cvetič, and references therein, *Top quark condensation*, Rev. Mod. Phys. **71**, 513 (1999).
 - [2] Y. Nambu and G. Jona-Lasinio, *Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*, Phys. Rev. **122**, 345 (1961); *Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II*, *ibid.* **124**, 246 (1961).
 - [3] Y. Nambu, Enrico Fermi Institute Report No. 89-08 (1989), *Bootstrap Symmetry Breaking in Electroweak Unification*.
 - [4] V. A. Miransky, M. Tanabashi and K. Yamawaki, *Dynamical Electroweak Symmetry Breaking with Large Anomalous Dimension and t Quark Condensate*, Phys. Lett. B **221**, 177 (1989).
 - [5] V. A. Miransky, M. Tanabashi and K. Yamawaki, *Is the t Quark Responsible for the Mass of W and Z Bosons?*, Mod. Phys. Lett. A **4**, 1043 (1989).
 - [6] W. J. Marciano, *Heavy Top Quark Mass Predictions*, Phys. Rev. Lett. **62**, 2793 (1989).
 - [7] W. J. Marciano, *Dynamical Symmetry Breaking and the Top Quark Mass*, Phys. Rev. D **41**, 219 (1990).
 - [8] W. A. Bardeen, C. T. Hill and M. Lindner, *Minimal Dynamical Symmetry Breaking of the Standard Model*, Phys. Rev. D **41**, 1647 (1990).
 - [9] D. W. Jung, O. C. W. Kong and J. S. Lee, *Holomorphic Supersymmetric Nambu-Jona-Lasinio Model with Application to Dynamical Electroweak Symmetry Breaking*, Phys. Rev. D **81**, 031701 (2010) .
 - [10] W. Buchmüller and S. T. Love, *Chiral Symmetry and Supersymmetry in the Nambu-Jona-Lasinio Model*, Nucl. Phys. B **204**, 213 (1982).
 - [11] W. Buchmüller and U. Ellwanger, *On the Structure of Composite Goldstino Supermultiplets*, Nucl. Phys. B **245**, 237 (1984).
 - [12] O. C. W. Kong, *Exploring an Alternative Supersymmetric Nambu-Jona-Lasinio Model*, AIP

- Conf. Proc.* **1200** (2010) 1101.
- [13] G. Faisel, D. W. Jung and O. C. W. Kong, *Dynamical Symmetry Breaking with Four-Superfield Interactions*, JHEP **1201**, 164 (2012).
 - [14] D. Yan-Min, G. Faisel, D. W. Jung and O. C. W. Kong, *Majorana versus Dirac Mass from Holomorphic Supersymmetric Nambu–Jona-Lasinio Model*, Phys. Rev. D **87**, 085033 (2013).
 - [15] See for example D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, *The Soft Supersymmetry Breaking Lagrangian: Theory and Applications*, Phys. Rept. **407**, 1 (2005).
 - [16] Y. Cheng, Y. M. Dai, G. Faisel and O. C. W. Kong, *A Simple Model of Dynamical Supersymmetry Breaking with the Generation of Soft Mass(es)*, arXiv:1507.01514 [hep-ph], *submitted to Phys. Rev. Lett.* (2015).
 - [17] J. Wess and J. Bagger, *Supersymmetry and Supergravity*, Princeton, USA: Univ. Pr. (1992)
 - [18] W. A. Bardeen, C. N. Leung and S. T. Love, *The Dilaton and Chiral Symmetry Breaking*, Phys. Rev. Lett. **56**, 1230 (1986).
 - [19] C. N. Leung, S. T. Love and W. A. Bardeen, *Spontaneous Symmetry Breaking in Scale Invariant Quantum Electrodynamics*, Nucl. Phys. B **273**, 649 (1986).
 - [20] C. N. Leung, S. T. Love and W. A. Bardeen, *Aspects of Dynamical Symmetry Breaking in Gauge Field Theories*, Nucl. Phys. B **323**, 493 (1989).
 - [21] S. Weinberg, *Perturbative Calculations of Symmetry Breaking*, Phys. Rev. D **7**, 2887 (1973).
 - [22] R.D.C. Miller, *A Tadpole Supergraph Method for the Evaluation of SUSY Effective Potentials*, Nucl. Phys. B **228** (1983) 316.
 - [23] See, for example, Z. Komargodski and N. Seiberg, *From Linear SUSY to Constrained Superfields*, JHEP **0909**, 066 (2009). The paper is particular relevant here as it discusses issues with the Goldstino in a more general setting. It will be interesting to see if one can learn something more about our model in that light.
 - [24] T. E. Clark, S. T. Love and W. A. Bardeen, *The Top Quark Mass in a Supersymmetric Standard Model with Dynamical Symmetry Breaking*, Phys. Lett. B **237**, 235 (1990).
 - [25] S. Ferrara, L. Girardello and F. Palumbo, *A General Mass Formula in Broken Supersymmetry*, Phys. Rev. D **20**, 403 (1979).
 - [26] M. T. Grisaru, M. Rocek and A. Karlhede, *The Superhiggs Effect in Superspace*, Phys. Lett. B **120**, 110 (1983).

- [27] P. Binétruy, *Supersymmetry: Theory, Experiment, and Cosmology*, Oxford (2006).
- [28] For a review, see Y. Shadmi and Y. Shirman, *Dynamical supersymmetry breaking*, Rev. Mod. Phys. **72**, 25 (2000) .
- [29] O.C.W. Kong, invited talk presented at the **FLASY 13**, Niigata, **Japan**, Jul 1-5, 2013, *NCU-HEP-k059*.
- [30] M. Suzuki, *Dynamical Composite Models of Electroweak Bosons*, Phys. Rev. D **37**, 210 (1988); *ibid. Approximate gauge symmetry of composite vector bosons*, Phys. Rev. D **82**, 045026 (2010).
- [31] O.C.W. Kong, NCU-HEP-k064, invited talk presented at the XS 2015 conference, July 6-8, 2015, Hong Kong.
- [32] H. K. Dreiner, H. E. Haber and S. Martin, *Two-component Spinor Techniques and Feynman Rules for Quantum Field Theory and Supersymmetry*, Phys. Rept. **494**, 1 (2010).